Chapter 5:
Real and Un-Real Options Analysis:
Theory of Option Pricing and Application to Real Assets

Introduction

For the past twenty years, virtually any book or article that addresses valuation, project finance, merger and acquisition analysis or just about any other financial topic includes a section or a chapter on how to value options. After reading the option chapter in one of the texts, you can probably skip chapters in most other books, because the writing tends to be very similar and there is rarely much insight in terms of how options analysis can mechanically be implemented in a realistic manner when making valuations. Applying option analysis in capital budgeting and valuation of non financial assets began not long after Black and Scholes published their famous article on quantifying the value of a financial option. This notion of real options implied that standard valuation models in which future cash flow is discounted for risk can undervalue investments. The undervaluation supposedly occurs because through actively managing investments throughout their life, downside risk can be avoided while upside potential can be obtained; meaning that the cash flow pattern distribution of an investment is not symmetric. For example, when deciding whether to make large repairs on a project in the middle of its life, if market conditions have turned out favorable, then the additional investment should be made. On the other hand, if the market has turned out to be below expectations, the decision to make the mid-life capital investment can be stopped and the project retired. By actively managing assets (in this case through deciding not to make the additional mid-life investment) the upside potential for returns is maintained while the downside is avoided. When applying the discounted cash flow model where base case cash flows are discounted at a rate that reflects the volatility of cash flows (i.e. upside and downside risk) the value of being able to manage the assets is not captured. This chapter addresses practical issues regarding how to quantify the value of real options and whether all of the fuss about real options is really warranted.

There a few reasons that so much discussion of real options occurs in academic texts and financial analysts seem to like to comment on “option plays” so often when speaking on television. First, the mathematics behind option pricing models is more sophisticated than the standard discounting of cash flows and therefore option pricing models seem to offer more intellectual stimulation – you can show how smart you are by mention option terms such as delta and gamma. Second, option pricing concepts can provide an effective way to confuse things and justify values that are higher than simple discounted cash flows would suggest. Given the general desire for people who want to make an investment to come up with high values, the option models imply the value is really higher instead of simply acknowledging that investment analyses are biased by vested interest. Third, when applying option pricing models, there is no need to use the CAPM that has so many practical and theoretical problems, but instead, you can calculate a single number that does not depend on inputs that are virtually impossible to measure. Despite all of the lovely theory and elegant results given by option models, in practice real option analysis has not taken hold in the way many expected it to a decade ago and option price techniques are not applied very often to valuation of real assets.

Sophisticated mathematical models that compute the price of option contracts -- the most famous of which is the Black-Scholes equation -- have been developed and refined in the last twenty-five years to value a wide variety of futures, options and other derivative products. Nowadays virtually anybody with an

MBA degree has at least plugged the strike price, volatility, risk free rate and the term to expiration into the Black-Scholes equation to find the value of an option. Given the elegance of the formula, economists, consultants and business school professors have attempted to frame a wide variety of business and other problems ranging from the timing of investment decisions to suicide as option analysis.[1] Notwithstanding the popularity of option pricing models, application of real options in practice is not easy because real assets do not have precisely defined strike prices, because managers to not often exercise options exit a business in an optimal manner, because cash flows produced from actual investments do not follow the same random walk patterns as stock prices, and because construction of financial models that incorporate flexibility can be a complex undertaking. Using examples from various different industries, this chapter describes how to build real option models as well as the type of implicit assumptions that must be made for real options analysis to produce valuations that are much different than more traditional valuation techniques. In working through real options analysis, the discussion demonstrates what effects on value occur from factors such as mean reversion, changing volatility, non-normal distributions and flexible strike prices.

Exercises with Option Pricing Models

Standard asset valuation techniques that discount future cash flows make an implicit or explicit assumption that the potential dispersion around the expected cash flow is not skewed – the downside risk is roughly similar to the upside potential. If, on the other hand, cash flows have a higher probability of exceeding the expected case because of limits on downside cash flows through management action, the standard discounted cash flow models do not present an accurate picture of the value of an asset. These skewed distributions can often be quantified using option pricing techniques. In using various different examples to describe how application of option pricing models can affect valuation analysis of capital intensive assets the chapter begins with review of financial option pricing concepts and explains how analytical models can be used to value the right not to operate a plant. Next, the contractual terms of options contracts are defined and standard option models are used to value financial instruments with high volatility and mean reversion. Once financial option models are explained, alternative valuation models are used to quantify various different real options.

Both the Black-Scholes equation and Monte Carlo models are used to compute the value of option contracts and plant investments. Since the Black-Scholes model is so often applied in practice, some exercises and mechanics of implementing the model are presented in appendix 1 of this chapter. In addition to working through how to program the model in a spreadsheet, the appendix contrasts valuation from the Black-Scholes model with results from Monte Carlo simulation. This exercise demonstrates that the structure of contracts as well as the high volatility and the extreme mean reversion limits the applicability of the Black-Scholes equation and other so-called closed from equations for option valuation. The second exercise involves real options and values a hypothetical investment assuming a series of different real options ranging from the option to expand; the option to delay construction; the option to cancel the investment during construction; and, the option to retire or mothball the plant. This exercise demonstrates that real options models add much less of a premium to the value of assets when mean reversion and undefined exercise prices are included in the analysis. It also demonstrates that the difficult part in valuing an option is often setting up a flexible financial model that can simulate management decisions. The exercise demonstrates that in many cases, even if the volatility of cash flow is quite high,

[1] Using option pricing models has even been suggested to explain the economics of committing suicide. See Pindyck, Robert and Dixit, Avinash, “Investment Under Uncertainty”, Princeton University Press, 1994, page 15. The authors suggest that if expected values are used suicide would occur when there are more days when expected value of living during the day is negative. However, if there is volatility and the chance for good things to happen, the value of the option suggests the expected value approach overstates the amount of suicide.

the real option does not change the valuation much as compared to classic discounted cash flow valuation. The final two cases apply Monte Carlo simulation to valuation of a real option two different types of real options. The first is the real option associated with the possibility of re-financing a toll road project and the second is the flexibility of making small rather than large investments to meet anticipated increases in demand.

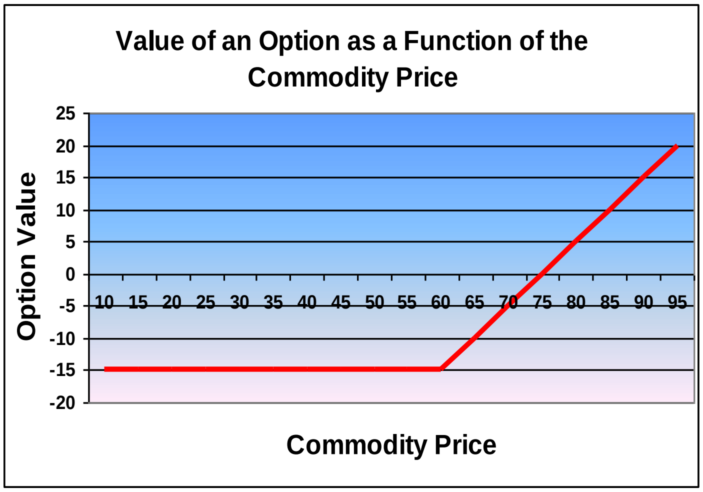
Option Contracts

For the sake of providing a comprehensive discussion, this section reviews the general structure of call and put options. If you are already familiar with option payoff graphs, you can skip the section without missing out on any of the ideas involving the value of real options. The classic definition of an option is that it gives you the right to do something, but you are not obligated to take the action. An option is also termed a form of a derivative because the value of an option is derived from the underlying value of the investment. In mathematics, a derivative measures the change in a variable as a function of another variable; similarly, the value of a financial derivative changes as a function of it another related financial variable. For example, a call option on the stock price of Microsoft changes as the underlying price of Microsoft's stock changes. Financial derivatives in general and option contracts in particular allow parties who wish to avoid taking risk in the underlying security to transfer risk to parties who are willing to be paid for taking on the risk. Financial derivatives have experienced a dramatic growth over the past few decades as illustrated by the volume of interest rate swap transactions (a financial instrument whose value is derived from changes in the level of interest rates.) The first interest rate swap transaction occurred in 1981. Two decades later the global swap market exceeded $5 trillion. In addition to stocks, interest rates and currencies, derivatives are traded on a wide variety of commodities ranging from electricity to palm oil to corn.

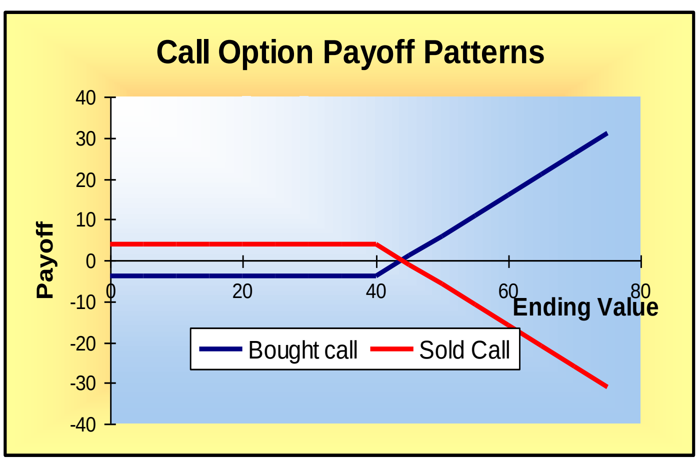
A formal option contract on a financial asset includes a strike price which is the fixed price at which an asset can be purchased or sold. It also contains a time frame that defines the period over which the right to buy or sell can be exercised. In its most general form, a call option is the right to act on information in the future that can provide the holder with an increase in value but to right not to commit to a decision means the payoff from a call option has a limited downside. Since downside risk is limited, the ultimate profit payoff from holding an option depends on future uncertainty associated with the value of the asset. This payoff structure is illustrated by familiar diagrams of the option profit as a function of the underlying uncertain price at the date the option is exercised. These diagrams plot the payoff of a derivative on the vertical axis given the possible uncertain future price outcomes on the horizontal axis. If units of the payoff are the same as units of the underlying asset, then a general characteristic of these payoff graphs is that there is a forty-five degree line showing that at some point value of the option increases or decreases in proportion to the underlying price.

The graph below represents a call option on a commodity where the owner of the option has the right to buy a given amount of the commodity at some point in the future at a fixed strike price. The vertical y-axis on the graph is the payoff from the call option at the date the option can be exercised, and the horizontal axis is the price of the commodity at the date the option can be exercised. The graph assumes a strike price of 60, meaning that the option has no value if the commodity price is below 60 because when the option expires, the fixed price that must be paid to buy the asset (say 50) exceeds the actual price of the asset. You wouldn't buy something for 60 when you could buy it for 50. However, as soon as the asset value increases to more than the exercise price, the holder will exercise the option and buy the commodity. For example, if the asset price is 61, then the asset can be purchased for 60 and immediately sold for 61 yielding a payoff of 1. If the price increases to 62, then the value received at the expiration date is 2. Therefore, if the actual price is above the strike price, the profit payoff on the option increases on a dollar for dollar basis as the price increases. The negative payoff of 15 when the price is below 60

and the option is not exercised simply reflects the cost of buying the call option or the premium. This is referred to as the option premium, the option price or the value of the call option in the discussion below.



For traded financial options, there is a buyer for every seller. This means that the payoff pattern for the seller of an option is the inverse of the pattern for a buyer. Whereas the buyer of a call option cannot lose more than the premium or the value of the option if the price at expiration turns out to be less than the exercise price – 60 in the above graph, the seller of a call option has a potential big downside if the price increases. If the value of the investment is above the strike price shown on the x-axis, then the seller must pay the difference between the strike price and the current value to the buyer of the call option. If, on the other hand, the price turns out to be less than the exercise price, then the seller earns the premium – the value of the option – without paying out anything. The seller of a call option compared to the buyer of a call option is illustrated on the graph below.

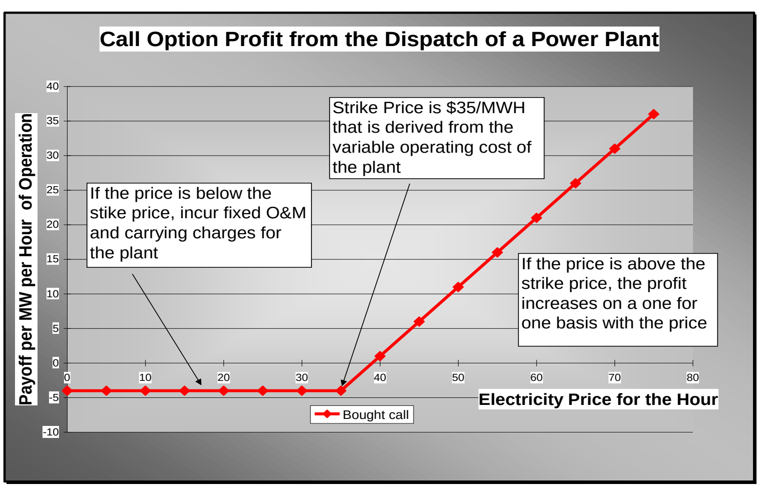


A put option has a different pattern from a call option because the put option contract in the case of the put option, the option gives the holder the right to sell rather than to buy. If the price turns out to be less

than the strike price, then one can sell the stock for the strike price and buy it for a lower value an realize a profit. Therefore, the payoff from buying a put option increases when the final realized price is below the strike price rather than above the strike price. In the last chapter, a put option was discussed in terms of the equity and debt claims on free cash flow. Debt is like selling a put option because when the value is lower than the strike price, losses occur and when the value is higher a fixed premium is made.

Any physical investment includes a number of options, most of which involve the right to exit the investment at one stage or another. These options include the right to cancel construction, the right to delay making investments, the right to not expand capacity, the right to retire at different dates and the right to not operate. To see how an investment can be analogous to a call option, recall the example of the electricity plant discussed in Chapter 1. Once the plant begins operation, the management has the right not to operate the plant. This management discretion can be characterized as a call option because the value of the plant in any hour depends on uncertain market prices. The dispatch decision is driven by the variable operating cost of a plant -- the exercise price -- and the market clearing electricity price (the uncertain future price.) If the electricity price is above the variable operating cost, the plant is dispatched -- the option is exercised. If the plant operates at its full capacity, the value realized by the plant in an hour increases in direct proportion to increases in the electricity price. On the other hand, if the electricity price is below the variable cost, the dispatch option is not exercised. Here, the plant must still pay fixed costs -- operation and maintenance and capital costs which can be defined as the option premium. When the option to dispatch is not exercised, the negative value from incurring fixed costs -- the option premium -- does not change no matter how low the price. As with the financial option, if the price is above the exercise price, the value continues to increase, but the downside effect of low prices is limited to option premium. If the dispatch option is correctly valued, then the expected value from the positive payoff when the option is exercised equals the cost (the premium) of the option.

The similarity of a call option on a financial security to the right of an electricity plant to dispatch is illustrated on the graph below. Here, the option price or the option premium is the fixed cost of the plant, the exercise price is the variable operating cost and the underlying price is the price of electricity.



Other options associated with management discretion can be illustrated in a similar manner. For example, the value of the cancellation option during construction is illustrated by showing the present

value of plant on the horizontal axis for a given period. Before construction is started, the costs to develop the plant in terms of acquiring permits, arranging loan agreements, paying for feasibility studies and writing up engineering plans can be considered the option premium. After the development occurs, there can be a range of values depending on the expected construction cost and future cash flows. The vertical axis shows the potential net present value of the plant. The strike price is the point at which the net present value is positive. This option is more valuable the longer the construction period and it declines as the commercial operation date approaches. In the case of the retirement option, assume that capital expenditures must be made to continue operating the plant after twenty-five years. If prices are relatively high, the capital expenditures will be made, while if the power prices are low, construction of the plant should be cancelled. As with the other options, the downside is limited and the upside increases with the uncertain cash flow.

The Lifetime of a Real Asset and Options to Exit the Investment

To consider real options that can occur over the lifetime of an investment, consider the analogy between an investment and a romantic relationship. A relationship usually has various stages beginning with meeting at a bar, going on a few dates and then gradually increasing the level of commitment culminating in an engagement and a marriage ceremony. After the marriage date, exiting the relationship becomes much more expensive to exit, especially with the birth of children. Eventually, the marriage can become more volatile and investments in marriage counseling and other services may be necessary to keep the relationship from ending. In a similar manner to the relationship, an investment has various stages which involve increasing levels of commitment and increasing costs of exit. The investment begins with the development phase where feasibility of the project is researched and contracts are drafted. Even though the development phase can take a long time, at this stage the financial commitment is relatively small and it is fairly easy to exit the investment. This is akin to the dating phase of a relationship when people are gathering information about each other. During the process of dating, the time period before which the next stage – engagement – can be extended. For the investment, the option of delaying construction of the asset or increasing the size of the investment can be exercised which increases the value of making expenditures to develop a project. After making the commitment to become engaged, the relationship can still be ended, but this comes along with more cost (for example the value of the engagement ring is typically lost by the man despite legal rules to the contrary.) In the case of an investment, once construction has started, the cost of canceling the investment still exists, but is more expensive and it may be difficult because management has an emotional attachment to the project. After the wedding the relationship can be cancelled through a divorce, but at a much higher cost that includes lawyer fees. Similarly, after the commercial operation date, there is a large financial commitment and canceling investment is more expensive. Finally, by the time the children have grown, the marriage may be somewhat stale and can be canceled. Refurbishing the relationship can involve costs such as psychologists, and overseas trips. This corresponds to the option to retire or refurbish the investment.

The above analogy between an investment and a relationship illustrated that real options associated with an investment generally involve the possibility of exiting or canceling. If management has a psychological attachment to the investment, exercising the options to exit the investment may be difficult, implying that the supposed value of management flexibility options does not in fact exist. For most real options to have value, management must be willing to acknowledge it has made an error. For example, in the middle of constructing a large refinery, if market conditions change – perhaps prices fall and demand declines, it will be very difficult to cancel the project, write-off the investment and admit that with hindsight construction was a bad idea. Instead, management will have a natural propensity to hire consultants who forecast increasing demand and increasing prices. When the plant is complete, exercising an option to cancel the plant will also be difficult. After spending many years developing, constructing and managing a plant, it will be difficult for even the most cold-hearted manager to evaluate the real option in an unbiased way. All of

the analytical discussion and indeed the general academic writing on real options assume that companies will make the decisions to exit investments in an unbiased manner. To the extent that valuable options are not exercised, the value of options is overstated.

Real options associated with an exiting an investment can be categorized according to the stage of an investment. After discussing some general issues associated with calculating the value of options, a process of valuing various of these options and evaluating the importance of the option in the context of the overall investment value is described:

- Abandon a project during the research or development phase
- Choose not to expand the size of an investment even though the expansion was originally expected
- Delay construction of a project after the development phase
- Abandon an investment during the construction phase
- Temporarily cease operations of a plant when prices fall below short-run marginal cost
- Retire or mothball a plant before the end of its physical life
- Extend the life of a plant by making capital expenditures instead of retiring the plant

Other options that can affect the value of an investment involve how it is financed. These options range from the well known fact than investment in any equity security has characteristics of a call option because the downside exposure of equity is limited by others – lenders, workers, suppliers and so forth – taking some of the downside risk associated with the investment. (Chapter 4) Similarly, any loan made to an investment has characteristics of a sold put option as the upside is limited and the downside can be as mush as the outstanding balance of the loan. More subtle and perhaps more interesting options are those associated with re-financing an investment, with structuring management incentives in an acquisition through including equity kickers in subordinated debt. Unlike the physical options that all are generally call options and involve exiting an investment, these options related to financing may be put or call options and they do not involve exiting an investment.

Freeport McMoran Case Study

Before discussing the detailed mechanics of how to incorporate real options into financial models, consider the case study of an actual investment. In teaching project finance courses to business school students at Harvard University and elsewhere, the case study of Freeport McMoran is used as to discuss risk and cost of capital.[2] Students are supposed to observe how the capital structure changes over the life of an investment, evaluate whether equity cash flow or free cash flow should be used making the investment decision and think about the appropriate cost of capital for the investment. These issues completely miss the real point of the story that were central to the investment decision. The Freeport case involves evaluation of the decision to initially develop the Fitchburg copper mine in Indonesia in the 1960's where infrastructure had to be developed and a host of construction risks, political risks, and operating risks were present along with the highly volatile price of copper. The project turned out to be a stunning financial success (although both the project and the company have been and continue to be very controversial from other perspectives in terms of taking resources like copper and gold out of the ground of developing countries while making huge profits; using private armies to protect assets; potentially assisting in assignation attempts of Fidel Castro and destroying the environment.) By Western standards, the success of the company is evidenced by Henry Kissinger being on the board of directors and Jim Kramer often shouting about the company on CNBC as much as the dramatic increase in shareholder value that occurred from the investment in Indonesia.

[2] Southport Case

When thinking about the original investment decision to develop the Fitchburg copper mine, it is difficult to imagine that the most important consideration in going ahead with the investment was some kind of archane model that measured the cost of capital and how the cost of capital should be adjusted for the capital structure. It is more plausible to think that the people making the decision would examine a series of scenarios and evaluate what ranges in the equity rate or return could be realized. In considering the scenarios, evaluation of potential upside opportunities and downside risks was, certainly with hindsight, more important than attempting to make forecasts of extremely volatile copper prices. Some of the upside potential and limited downsides, which could be labeled as real options include:

- The option to stop after development and to make to make more expenditures to test the amount of ore (the risk of the project not having sufficient reserves could be mitigated by increased exploration rather than attempting to measure the cost of capital);

- The option to delay construction of the project if copper prices were low (the project was being considered because copper prices had increased) presumably it could have been constructed earlier if copper prices had risen earlier.

- The limitation of downside risk to equity holders (equity was only 17% of the project financing) which was accepted by lenders and the potential for high returns is the project was successful. This is the normal put option characteristics of a debt contract.

- The use of political risk insurance to mitigate political risks (rather than incurring a higher cost of capital, the risks could be directly quantified and paid for by insurance which is like buying a put option.)

- The option to re-finance the investment (with prepayment, the debt was scheduled to be paid off before the project was complete; if the project was to be successful it could be re-financed which means it could achieve an even higher return on equity.)

- The option to close mines if prices fall below variable cost of production (if the variable cost of production is near the price, the upside is larger than the downside because the mine can be shut down.)

- Most importantly, the option to expand the size of the project (the revenues from the first stage of the mine were a very small fraction of the revenues actually realized because Freeport was able to expand the mine to ultimately be the largest open pit mine in the world.)

The important thing about this story is not to attempting to quantify the options through some kind of Monte Carlo simulation which assumes that copper prices follow a normal distribution or to try an apply the Black-Scholes formula in the analysis. Instead, if the return on equity were any near reasonable for the project (perhaps above the risk free rate), then the potential for upside is far higher than the exposure to downside risk. Other investments such as investments in sub-prime loans, contrasted dramatically with the Freeport example. Here the downside risk was far greater than the upside potential and there was no flexibility to change anything once the downside case occurred. The true benefit of thinking about real options is that one should examine potential upsides and downsides rather than wasting too much time on the cost of capital.

The Option to Delay an Investment

In the mid-1990's two economists named Dixit and Pindyck wrote a book named "Investment Under Uncertainty" which described how real options (or, if you want to sound more sophisticated, contingent claims) can be measured using sophisticated mathematical analysis.[3] They asserted that one of the most basic ideas in economics of equating marginal cost to price is highly affected by the real option to delay construction of an investment. The authors observed that an investment decision is not simply an either/or choice between making or not making an investment, but rather a decision that also can involve delaying construction of an investment and pulling the plug to make the investment. Dixit and Pindyck concluded that the delay option is very important in investment analysis and ultimately results in prices higher than marginal cost because there will be chronic underinvestment as companies will continually exercise the option to delay.

The idea of a real option to delay an investment involves foregoing potential positive cash flows after the investment could be made in order to gain more information about future cash flows. After waiting and observing what happens to potential cash flows, the investment can be re-assessed with the additional information. To illustrate the delay option, consider the following example used by Dixit and Pindyck:

- The cost the investment is 1,600;
- The cash flow realized from the investment is 200 for the current year;
- Cash flow in the subsequent year can either be 100 or 300 and it remains at that level indefinitely;
- There is a 50% probability of either the 100 or the 300 cash flow level; and,
- The discount rate is 10%.

With these assumptions, if the expected net present value of the cash flow is computed, the value is 2,000 which is 200 (the average of 100 and 300) divided by 10%. This value of 2,000 is greater than the investment cost of 1,600 implying that the investment should be made if standard net present value rules are applied as it has a positive net present value of 400 (2,000 minus 1,600.) While the decision to go ahead rather than avoid the investment may seem obvious, this decision ignores a third alternative which is to delay the project. If the project is delayed by one year, then the investment will not be made in the state of the world when the cash flow is 100 but it will go ahead when the cash flow is 300. Further, if the project is delayed, the benefits of realizing cash flow of 200 in the first year will not be realized. In the high cash flow state of the world, the value of the investment after one year delay in cash flow is 3,000. After subtracting the lost value of the 200 cash flow and accounting for the 50% probability of realizing the upside case, the value is ((3,000-1,600)/1.1) x .5 - 200 or 436. This value is higher than the value of 400 generated from the investment without a delay because the downside case cash flows are avoided. Therefore, the best alternative is not to make investment without delay; nor is it to cancel the investment for good. Rather, the option with the highest value is to delay the project for a year.

While this example appears to demonstrate that the delay option can be valuable, when selected assumptions are changed, the benefits of the option become much more ambiguous. If prices are not resolved to be either 300 or 100 forever, but instead gradually converge to the mean level of 200, the value of delaying the option dramatically declines. In a case where the mean level of 200 is attained after two years (after the value of 300 or 100), the value of the prospective cash flow in the upside case is 2,173 rather than 3,000. This means that the net value in the upside case falls to 87 after accounting for the lost opportunity of the 200 cash flow in the first year. (The 2,173 is computed by discounting the first 300 cash flow by 1.1, the second cash flow of 300 by 1.21 and the remaining value of 2,000 -- 200/.1 -- at 1.21.) Even if it takes ten years for the cash flows to revert to the mean level, the value of delay is still less than the value in the case where the option to delay is not selected.

[3] Dixit, A. and Pindyck, R. 1994. *Investment under Uncertainty,* Princeton, NJ: Princeton University Press.

A second problem with the example is the artificial assumption that there is no possibility that cash flows will remain at 200, but that they will suddenly become either much higher or much lower. If there is a possibility that the cash flow will remain at the current levels, and then later diverge to either 200 or 300 in the third year, then, after the first decision is made to delay the project, another decision should be made to exercise the delay option again. If the delay option is exercised over and over again, the investment never gets built and the delay option becomes a cancellation option. In this situation, as all firms would make the same delay decision, the price ultimately will increase with underinvestment. But then, more projects would be built because of they become more profitable the increased price. In this example, the option to delay creates itself mean reversion in cash flow. Therefore, the argument that prices remain above marginal cost because companies exercise the option to delay is not consistent with basic economic principles. In sum, seemingly obvious benefits of exercising the delay option become much more questionable when more complex assumptions are made.

Valuation Analysis of Real Options

By the late 1990's the idea of using real options in valuation of investments had become a hot topic for financial analysts in many different industries. Tom Copeland along with Vladimir Antikarov, who had earlier co-authored a well known book on Valuation using DCF, wrote a second book in 2001 named "Real Options, a Practitioner's Guide."[4] The book made real options analysis almost sound cult-like and implied that real options analysis could dramatically change way investments are valued. For example, the authors assert that "in ten years, real options will replace NPV as the as the central paradigm for investment decisions." At the time that real options were in fashion, seminars were often given on real options and academics suggested that real options could explain inconsistencies in the CAPM.

The question of how much real options analysis can change the valuation of an investment relative to traditional net present value analysis is considered in the paragraphs below. To address this issue, the details of hoe financial models can be modified to reflect management flexibility is discussed along with how to use Monte Carlo simulation to measure how different volatility and other time series parameters influence the value of real options. This analysis is applied to a variety of different real options including the option to not proceed with a project after development; the option to cancel a project during construction; the option to retire a project earlier or later than its economic life; and the option not to expand the size of the project. For each of the options, the increment in value from the option is measured using different levels of volatility and different levels of mean reversion and price boundaries in order to quantify the relative importance of the option.

Unlike many writings on options, this chapter concentrates on practical issues of how to add flexibility into financial models. When presenting a series of real options that could be applied to all sorts of investments ranging from real estate to telecommunications below, the models use very simple inputs with respect to cash flow, capital expenditures and development expenditures. Through keeping the operating aspects of the examples simple, the idea is that the models have general implications across industries.

The Option to Expand

The option to expand a business endeavor is one of the primary ways in which people who advocate real options suggest that standard discount cash flow understates the value of investments. In the Freeport case discussed above, the rate of return and the net present value on the initial mine investment that was

4 Copeland, T. and V Antikarov, *Real Options a Practitioner's Guide,* 2001, Texere, N.Y.

made in the 1960's was all not very important in the overall scheme of things. What allowed the company to put Henry Kissinger on its board of directors was the dramatic expansion of the project in later years well after the end of the life of the initial project. Without the first investment, Freeport would not have gotten a foothold in Indonesia and been able to become one of the largest mining companies in the world. With hindsight, it was the option to expand the project that overwhelmed the value of anything else in the initial investment period. A very similar process is often used in discussing software projects where the return on the first version of the software may be less important than opportunity to make much bigger profits on the second version. For example, the profits Microsoft made on DOS operating system in the 1980's were very good; but the real value of this investment, that made Bill Gates the richest man on the planet, was the potential to expand to the windows systems later on.

The expansion option is presented as the first real options because, from a financial modeling mechanical perspective, the option to expand an investment is fairly easy to construct. In building a financial model that includes an option to expand, one can build a model with two stages of investment. Without considering the value of the option, the first stage of the investment may be valued or alternatively, the first and the second stage together can be assessed. With the option, the investment is assessed by valuing the first stage of the investment and then only including the second stage of the investment in the cash flows for valuation if the expansion turns out to be profitable. In quantifying the second valuation approach that includes the option to expand, a model must be created that changes the decision to make the expansion investment as a function of the expected level of cash flows that will be realized at the time the second investment must be made. Since the value of the option to expand depends on uncertainty associated with cash flow, the model needs to also incorporate potential dispersion in cash flow that drives the investment value; otherwise if there was no uncertainty in value, the decision with respect to expansion would always be the same. In explaining how to compute the value of the investment with an option, the mechanics of developing a model with flexibility are first presented without simulation of the dispersion in cash flow. Next, a time series equation is included in defining cash flow and Monte Carlo simulation is added to the model. Finally, the model is used to analyze the value of the option using alternative parameters with respect to the structure of the investment and time series parameters.

The mechanics of creating a model that can incorporate an option to expand involve defining the timing of the expansion investment, the operating period of the first stage of the investment, the operating period of the second expansion stage of the investment and the growth rate in cash flow.5 With these inputs, switch variables are defined for the different investment periods, and the case flows are computed. Switch variables can be computed by defining the age of the project and using the AND function as described in chapter 2. Given that TRUE equals 1 and FALSE equals zero, the cash flow from the first stage and the expansion stage are computed using the equations:

$$1^{st}\text{ Stage Cash Flow} = 1^{st}\text{ Stage Operation Switch} \times \text{Cash Flow} - 1^{st}\text{ Investment Switch} \times 1^{st}\text{ Investment}$$

$$2^{nd}\text{ Stage Cash Flow} = 2^{nd}\text{ Stage Operation Switch} \times \text{Cash Flow} - 2^{nd}\text{ Investment Switch} \times 2^{nd}\text{ Investment}$$

Inputs for the model and the layout of cash flows using the switch variables are illustrated in the table below.

5 An exercise that walks through the mechanics of creating this model and subsequent models is included on the CD.

Inputs

Cash Flow	90
Cost of Initial Investment	500
Cost of Expansion	1,000
Cash Flow Growth Rate	10%
Life of First Stage	5
Life Extension from Investment	7
Hurdle Rate	8%

Model

Age	0	1	2	3	4	5	6	7	8	9	10	11	12	13
First Investment	TRUE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE
First Stage Operation Period	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE
Expansion Investment Switch	FALSE	FALSE	FALSE	FALSE	FALSE	TRUE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE
Expansion Operation Switch	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	FALSE
Base Cash Flow	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00
Growth Index	1.00	1.10	1.21	1.33	1.46	1.61	1.77	1.95	2.14	2.36	2.59	2.85	3.14	3.45
Applied Cash Flow	90.00	99.00	108.90	119.79	131.77	144.95	159.44	175.38	192.92	212.22	233.44	256.78	282.46	310.70
Cash Flow from First Stage	(500.00)	99.00	108.90	119.79	131.77	144.95	-	-	-	-	-	-	-	-
Cash Flow from Expansion Stage	-	-	-	-	-	(1,000.00)	159.44	175.38	192.92	212.22	233.44	256.78	282.46	-
Total Cash Flow	(500.00)	99.00	108.90	119.79	131.77	(855.05)	159.44	175.38	192.92	212.22	233.44	256.78	282.46	-

The value of the first stage of the project, the expansion stage and the combined first two stages can be computed once the cash flows are established with the given discount rate. The real options models work better if the decisions and outputs are the present value rather than the IRR because of the possibility of undefined IRR's when the cash flows are very low. Once the values for the first stage, the expansion stage and the total of both stages are determined, the decision to expand can be simulated by creating a variable that is either TRUE or FALSE depending on whether the value of the expansion stage of the investment is positive. A test variable can be computed using a test variable as illustrated by the equation below:

NPV(Second stage cash flow) > 0

Using this test variable, the total cash flow with the option is computed using this variable as follows:

Total cash flow with option = First stage cash flow x Test variable

With total cash flow including the option, the present value with the option can be compared to the value of the first stage as well as the present value of the combined first and second stage. The mechanical calculations of value are illustrated on the table below:

Valuation without Option

	IRR	Disc Rate	NPV
First Stage	6%	8%	($22.57)
Expansion Stage	10%	8%	$58.37
First Stage Plus Expansion Stage	9%	8%	$35.80

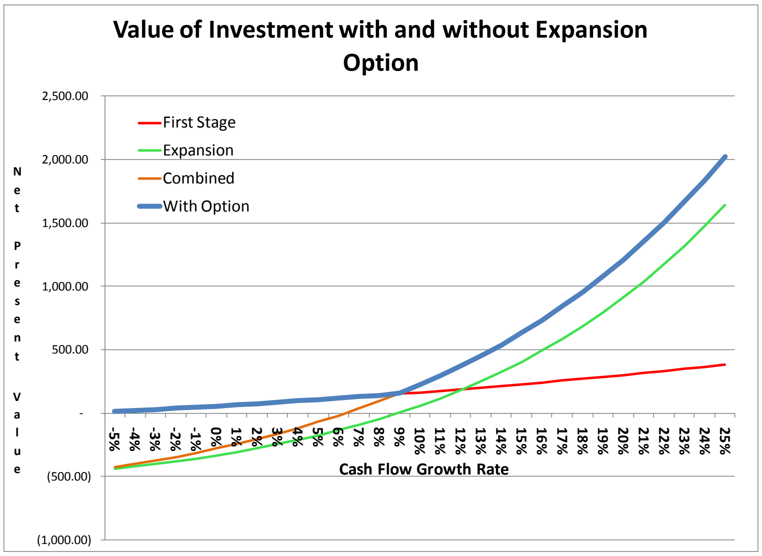
Option Valuation

Test for Expansion Investment	TRUE													
Cash Flow with Option to Expand	(500.00)	99.00	108.90	119.79	131.77	(855.05)	159.44	175.38	192.92	212.22	233.44	256.78	282.46	-

	IRR	Disc Rate	NPV
Value with Expansion Options	9%	8%	$35.80

Alternative values with and without the expansion option can be computed assuming different growth rates to demonstrate how the model simulates management flexibility. When the growth rates are low the option to expand is not exercised and when the growth rate is high the expansion investment is made. A scenario analysis of different growth rates is computed using a one way data table as described in chapter 3. To quickly make a graph of values in the data table, the top line of the data table can be temporarily hidden using the SHIFT,ALT,→ combination and the F11 key will lay out the data. The graph shows that if

the growth rate is low, the value with the option is the same as the value without the second stage and if the growth rate is high, then the value of the option is the same as the combined cash flow. In the graph below, when the growth rate is below 9% the expansion option is not exercised while when the growth rate is higher than 9%, the expansion investment is made. Note that the graph resembles the standard option payout graph shown in the section above. In this case, the size of the expansion investment is analogous to the strike price as if the investment is lower, then it is more advantageous to exercise the option at lower and lower growth rates in the example below. The premium paid for the option is amount of the first investment is analogous to the option premium. For example if the amount of the first investment is very high, then it is expensive to have the opportunity to exercise the expansion option.

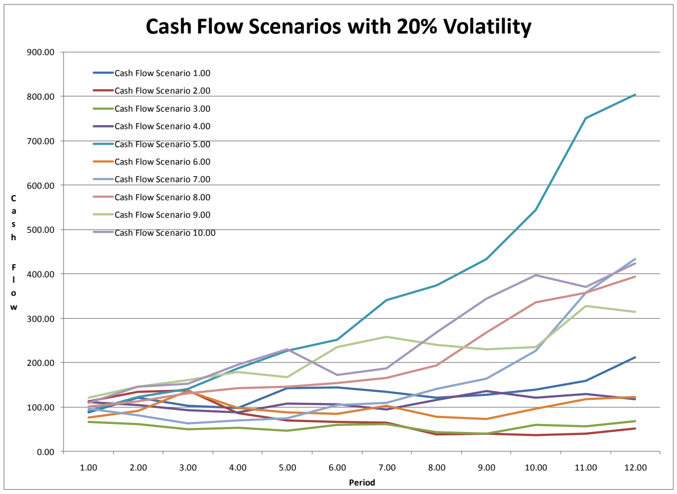


If one knew the growth rate with certainty, then the expansion option would either be selected or it would not be selected and the value would be established. Nothing else would have to be done. However, if there is uncertainty associated with the level of the cash flow, the option may or may not be exercised and the value of the option depends on the distribution of future cash flows. To compute the value of with Monte Carlo simulation, parameters such as volatility and mean reversion must be included as inputs. The reason a macro is useful here is that the Monte Carlos simulation can be structured to repeat calculations. Here, the macro works because each time the macro re-computes value using a The value of the option can be computed using Monte Carlo simulation as demonstrated using the equation below.

$$\text{Cash Flow}_t = \text{Cash Flow}_{t-1} \times (1+\text{Growth}) \times (1+\text{NORMSINV(RAND()} \times \text{Volatility})$$

To see how this equation works, consider the case when the volatility is zero. Here the cash flow is the same as it is in the case with no simulation. However when the volatility is positive and the random number is not equal to .5, the cash flow growth can vary by a wide margin as illustrated in the graph below which displays ten of the cash flow scenarios. With a high growth rate and a relatively long period, the volatility results in a wide dispersion of possible operating income over long periods. The concept of volatility was discussed in chapter 3 along with methods to compute volatility. If you an evaluating a new project, it is unlikely that you have any historic data or implied volatility statistics. Instead, you make a judgment as to what is a 68% for one standard deviation of 95% for two standard deviation range in growth rates. You can also then perform sensitivity analysis with respect to alternative growth. (To create

the graph shown below, you can create a one way data table with the cash flows on the top row and anything in the left column. Then use any cell for the column input.)



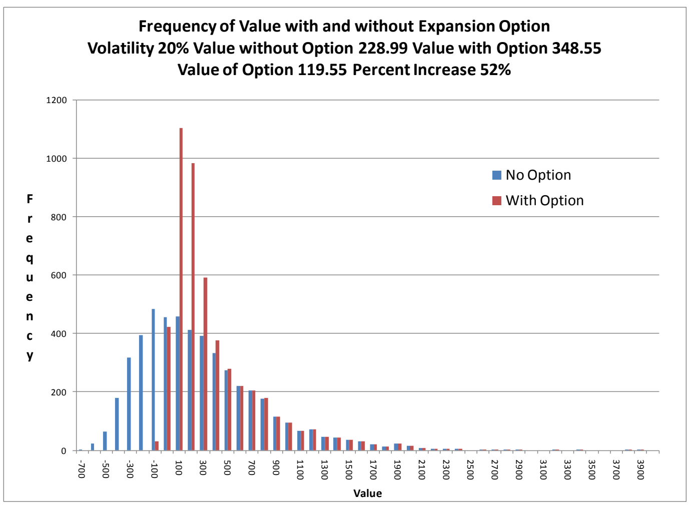
Using the different scenarios computed from the defined variation in cash flow, the various measures of value with and without the option can be developed. With some of the random variations, when the resulting operating cash flow is relatively high, the option to expand will be selected. In other scenarios where the cash flow is relatively low, the project will not proceed after the first stage. By computing the average value across the scenarios, one can compute how much the option to expand adds to the overall value of the project. To do this, a simple macro can be written to implement the simulation. Such a macro that makes a simulation using a FOR and NEXT loop and a cells command is demonstrated in the example below. It works because each time a new row is selected, a whole new set of random numbers is used creating a new simulation. (If you forget the range names, you can press the F3 key and then use the past list option. Then you can copy the list from the excel file into the macro.)

```
FOR ROW = 1 TO 10,000

        CELLS(ROW,101) = RANGE("PV_OF_FIRST_STAGE")
        CELLS(ROW,102) = RANGE("PV_OF_FIRST_AND_SECOND_STAGE")
        CELLS(ROW,103) = RANGE("PV_WITH_OPTION")

NEXT ROW
```

The simulation program above produces a series of 10,000 scenarios is printed in columns 101, 102, and 103 (you can of course begin with a different row and use different columns.) When the simulation is finished, the first step is to compare the average value of all the simulations without an expansion to the average value of all of the simulation options with the expansion option. The maximum of these two average values defines the value of the project without the option. This value can then be compared to the average value of the project which assumes there is management flexibility to proceed with the option to expand depending on the realized cash flow. Using the 20% volatility assumption and other assumptions with respect to the cost of the expansion, the expansion increases the value of the project by 52% as shown in the table below.

Summary of Expansion Option Value	
First Stage	164.98
Value with Expansion	228.99
Value without Option	228.99
Value with Option	348.55
Value of Option	119.55
Percent Increase	52%

Using values from each simulation, the frequency distribution can be summarized with a frequency graph (recall from chapter 3 that the FREQUENCY function can be used to create a graph by creating bins, selecting a target area, entering the function and finally using the SHIFT, CNTL, ENTER series of keys.) A frequency graph of the three series is shown on the graph below for the value with the option and the value without the option. The graph illustrates that when the option is included, the average of the values is higher than other cases and the distribution is skewed in a positive direction with a limit on the downside.



The analysis above has an implicit error because it assumes that when a manager makes the decision to expand the project, he has knowledge of future uncertain cash flows. For example, assume the volatility in cash flows means that the growth declines from 20% at the year of expansion down to 0% after the decision is made on whether or not to expand. Using the approach above, the expansion investment would not be made, as the realized cash flow is very low. However, at the time of the expansion decision, the growth rate was expected to be 20% and the expansion decision would be made. Therefore, the decision is made with information that is not available. To correct the model so that it only reflects information that is available, one can assume the project is sold at the date of the expansion and that when it is sold, the expected cash flow is the cash flow at the date of expansion plus the growth rate without any additional volatility. For example, if volatility during the first stage produces a cash flow that is lower than the expected case without volatility by 40% and the growth rate is 10%, then the prospective cash flow grows at 10% beginning with the 40% lower cash flow. When the correction is made that does not assume unavailable information can be used, the value of the option declines from 52% of the value

without the option to 43%.

In addition to volatility, mean reversion can be added to the analysis. As discussed earlier in the book, mean reversion adjustment moves the cash flow back to the expected level after volatility causes the cash flow to increase or decrease. Modeling mean reversion can be accomplished by adding a factor to the cash flow equation which has the following form:

Mean Reversion Adjustment = (Volatility Factor – 1) x Mean Reversion Factor

The mean reversion factor can be applied after the expansion period, as the idea of mean reversion is that volatility factors gradually move to toward 1.0. The manner in which stable volatility and mean reversion can be incorporated in a model are illustrated in the table below.

Model

Age	0	1	2	3	4	5	6	7	8	9	10	11
First Investment	TRUE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE
First Stage Operation Period	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE
Expansion Investment Switch	FALSE	FALSE	FALSE	FALSE	FALSE	TRUE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE
Expansion Operation Switch	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
Base Cash Flow	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00
Growth Index	1.00	1.10	1.21	1.33	1.46	1.61	1.77	1.95	2.14	2.36	2.59	2.85
Volatility Index Unadjusted	1.00	1.29	1.04	0.65	0.65	1.22	1.41	1.10	1.80	1.23	0.57	1.08
Volatility Index Adjusted	1.00	1.29	1.04	0.65	0.65	1.22	1.32	1.23	1.16	1.11	1.08	1.05
Mean Reversion Adjustment	-	-	0.09	0.01	(0.10)	(0.10)	0.10	0.07	0.05	0.03	0.02	0.02
Final Time Series Adjustment	1.00	1.29	0.95	0.64	0.76	1.32	1.23	1.16	1.11	1.08	1.05	1.04
Final Index	1.00	1.42	1.15	0.86	1.11	2.13	2.17	2.26	2.38	2.54	2.73	2.96

The first line on the table above uses the NORMSINV function and is not adjusted after the expansion period. It is defined on the basis of the final time series adjustment as follows:

$$\text{Volatility Index Unadjusted}_t = \text{Final Index}_{t-1} \times (1 + \text{NORMSINV(RAND())} \times \text{Volatility})$$

In order to hold the volatility constant after beginning the expansion stage, the next line uses the switch variables as follows:

$$\text{Volatility Index Adjusted}_t = 1^{st}\ \text{Stage} \times \text{Volatility Index} + 2^{nd}\ \text{Stage} \times \text{Adjusted Volatility}_{t-1}$$

The mean reversion adjustment is calculated as the using the adjusted volatility index so that it will apply in both the first stage and the expansion stage. In the first stage the mean reversion adjustment moderates the effect of volatility and moves the cash flow back to the expected levels. After the expansion, the mean reversion adjustment gradually moves the volatility factor back to a level of 1.0 as cumulative effects of the adjustment are gradually reduced since the volatility factor does not change with random movements. Once the mean reversion is computed using the equation described using the equation above, the final time series adjustment and the applied adjustment factors are computed using the following equations:

Final Time Series Adjustment = Volatility Index Adjusted + Mean Reversion Factor

$$\text{Applied Growth Index}_t = \text{Applied Growth Index}_{t-1} \times \text{Final Time Series Adjustment}_t$$

Using the model with the time series adjustments, a series of different sensitivity adjustments is shown on the table below. The first case assumes a mean reversion factor of 10%, implying that a random shock diminishes in ten years. As shown in the table, the mean reversion adjustment has a dramatic effect on

the value of the option as the value of option declines from 43% to 17%. The second case extends the life of the first stage by five years, which is analogous to lengthening the life of a call option. This case decreases the value of the option although it increases the overall value of the asset. increases the value of the option to ___%. The third case lengthens the life of the expansion option which is analogous to lowering the cost of exercising the option. This increases the overall value of the project, ____. The fourth case reduces the cost by of the expansion by 500 stage which is lowering the exercise price of an option meaning that it is more in the money.   first version may be marginally profitable, but developing the first version is the only way to proceed to a second version that produces a very high equity IRR. The first version provides an option to expand that can be exercised.

Development Phase Options

Development expenditures can be made for feasibility studies, evaluation of resources such as wind studies or oil reserve potential, research on a new drug, structuring contracts, evaluating different sites for locating a plant, lawyers for signing supply contracts, marketing research and other items. Specific examples of the development phase include exploration for oil, setting-up a high tech company or developing contracts and loan agreements for a toll road project. Costs are generally relatively small for the development phase, for example in project finance the expenditures are expected to range from 3% to 15% of the total cost of a project. However, the development costs are generally not financed by debt and there may be a large probability that project does not proceed after the development stage. Given that the project cannot proceed without development and the large possibility in some projects of not proceeding past the development stage, analysis of this seemingly minor aspect of the overall project can be an important issue.

One way to consider the value of expenditures for development is that they give you an option to proceed with the investment and realize positive cash flows. To see that the investment in development has a payoff structure similar to a call option, assume that an investment is being considered where the development cost is 5% or the total cost. Further assume that should the development be a success, the project can be sold for five times the development expenditure because it will have a positive net present value. In this example, the development expenditure is analogous to the call premium and the net value of the option is driven by the probability that the project can move beyond the development stage without being cancelled. Given that the downside is limited by the alternative to not go ahead with the project and the potential upside is higher the greater the volatility of the project, the payoff pattern is similar to the call option described above.

Modeling the value of the development expenditures depends on the nature of the development and how much of the uncertainty is resolved during the development period. If the development phase involves performing research or exploration that will give a much better indication of if the project will be a success, then the option should be valued using a decision tree where the key variable is the probability of success in latter stages. In this situation, the use of sophisticated mathematical analysis involving time series is largely irrelevant; the key issue is estimating probabilities of proceeding. On the other hand, if the development stage involves resolving uncertainty that is driven by external events such as market prices or varying demand, then the value gained from additional time is far lower and depends to a large extent on the amount of mean reversion in the economic variables that drive the value of the investment. Valuation of the two alternative types of development option involves different modeling techniques and is addressed separately in the next two sections. The former type of development is termed the research phase while the latter is labeled the evaluation and contracting phase to distinguish development options that depend on technology from development options that depend on economic factors.

Research Phase Cancellation Options

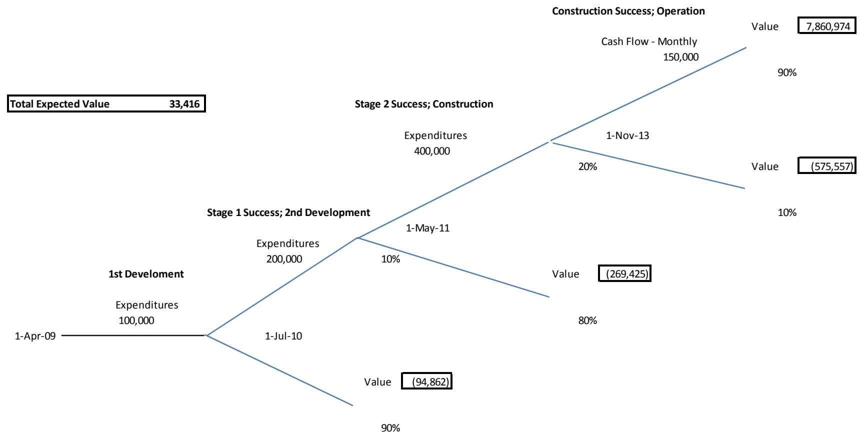
The value of the type of development option where some kind of research can resolve uncertainty is driven by estimates of the probability that the expenditures will allow the project to continue. The reason for making an investment in research is so the project can proceed to subsequent stages and ultimately perhaps realize a large upside if the project is a success. At each stage of the research, probability of success increases if the project is not cancelled. The key to valuing this type of research option is not making elaborate mathematical models that incorporate volatility or somehow trying to adapt the Black-Scholes model to the valuation, but rather obtaining accurate and unbiased estimates of the probability of success in each stage. Once probability estimates can be developed, the remaining issues involve setting a financial model to value the option and establishing an appropriate way to measure the discount rate applicable to cash flows.

To illustrate this type of development option, consider the investment in a geothermal electricity plant which has similarities to oil and gas exploration and analogies to development of a new drug in a pharmacetucial company. In the case of a geothermal plant, the key operational risk is the question of whether enough steam is available to produce electricity. Given this risk, the first step in the plant development is drilling some test wells. The value of making the investment in the test wells depends on the probability that the test will lead to a successful investment. To value a potential geothermal plant before the development stage, one could proceed in two different ways. The first way is to simply assume the development expenditure will be incurred in a similar manner as the construction expenditures and compute the IRR on the overall investment where cash flows are gauged against both the development expenditure and the construction expenditure. The second method is to explicitly account for the probability that the well exploration will be a success using a decision tree.

Assume that there are two stages of development and that the probability of the wells producing enough steam and proceeding successfully to the next stage is 10%. The second stage of development has a higher success probability of 20% as once the first set of wells are successful, it is more likely that the next set of exploratory wells will also be successful. If the second set of exploration wells is successful in producing sufficient steam, then construction proceeds and there is a 90% chance that the project will produce enough steam to produce electricity for 30 years. A diagram of assumed outcomes form the various development phases, the construction phase and the operation phase is shown below. To compute the value at each stage, the cost, the discount rate and the expenditure are required.

Assumptions

First Exploration	
Construction Start	1-Apr-09
Months of Development	15 Months
Cost of Development	100,000
Probability of Proceeding	10%
Discount Rate	8%
Second Exploration	
Start Date	1-Jul-10
Months of Development	10 Months
Cost of Development	200,000
Probability of Proceeding	20%
Discount Rate	8%
Construction	
Start Date	1-May-11
Months of Construction	30 Months
Cost of Construction	400,000
Probability of Proceeding	90%
Discount Rate	8%
Operation	
Start Date	1-Nov-13
Plant Life	360
Cash Flow	150,000
Discount Rate	12%
End Date	1-Nov-43



Setting up a financial model that can value the investment involves being able to quantify cash flows in different stages and separately quantifying and discount the cash flows realized in the different periods. This can be accomplished using a couple of spreadsheet tools. The first is establishing switches for each stage using the TRUE/FALSE logical switches analogous to the technique described in Chapter 2 for project finance models. The second is computing present values that incorporate the length of the stages and distinguish cash flows for each stage. This can be accomplished through multiplying the different TRUE/FALSE switches by present value factors using the SUMPRODUCT formula. The specifics of the calculation involve the following step by step approach:

- Input the costs of development, probability and length of each of the investment stages as illustrated in the diagram above.
- Compute the total amount of time (e.g. months) that occur between the first period of development and the date at which the project becomes operational. To do this, simply sum the number of months in each period of development and construction. (The EDATE function can be used to compute the start date of each stage shown on the above diagram.)
- The start date for the period code of the model is a negative number computed using the formula:

  1 - Total of Development and Construction Months

- Enter the start date and end date for each period by entering the initial date of the first development stage and then subsequent start and end dates using the EDATE function. If the periods are entered in months, the start date of the next date is computed using the formula:

  EDATE(prior stage start date, number of months of prior stage).

- Calculation of the start period of subsequent stages is a bit tricky. Begin with the construction phase – the last stage before operation, where the start period is 1-construction months, analogous to the start period of the entire project shown above. Then work backwards from the next farthest stage (the second development stage in this example) and subtract the number of months in this stage to the construction phase (you end up with a larger negative number.) After the period code is computed for the middle stage, make the same calculation for the subsequent stages until you compute the number for the first development stage.

Results of this type of analysis of the period codes are illustrated in the table below.

Time Period Codes for Start of Different Stages

	Period Code	Months
Total Pre-Commercial Months	55 Months	
Start Period	-54 Months	
Start of Construction Stage	-29 Months	30.00
Start of Second Stage	-39 Months	10.00
Start of First Stage	-54 Months	15.00
Finish of Operation Stage	360 Months	360.00

- Once the time period codes for the beginning of each stage are computed, use the AND function to compute switches for the various stages. Begin with the first development stage which is greater than or equal to the start of the first stage and less than the start of the second stage. The second stage is greater than or equal to the second start and less than the construction start period. An illustration of this process is shown on the diagram below which shows selected periods in where the phase of the project changes.

Period Code	-54.00	-53.00	-52.00	-41.00	-40.00	-39.00	-31.00	-30.00	-1.00	0.00
Months	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Start Date	1-Apr-09	1-May-09	1-Jun-09	1-May-10	1-Jun-10	1-Jul-10	1-Mar-11	1-Apr-11	1-Sep-13	1-Oct-13
End Date	30-Apr-09	31-May-09	30-Jun-09	31-May-10	30-Jun-10	31-Jul-10	31-Mar-11	30-Apr-11	30-Sep-13	31-Oct-13
Pre-Operational Stage	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
First Development Stage	TRUE	TRUE	TRUE	TRUE	TRUE	FALSE	FALSE	FALSE	FALSE	FALSE
Second Development Stage	FALSE	FALSE	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	FALSE	FALSE
Construction Stage	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	TRUE	TRUE
Operation Stage	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE

- After the switches are established, the cash flows can be computed by attributing the total expenditures to the time periods. If the distribution of the expenditures is equal for each month of the phase, the formula for the cash flows can be simply computed as:

  Development Switch x Total Development Cost/Number of Months in Stage

- Once the cash flows are established, the amounts must be discounted and segregated so that the accumulated value for each stem of the decision tree can be computed. Using the switches that define each stage of the project, the value can be computed through multiplying the cash flows by the relevant switch and then multiplying the product of the switch and the cash flow by present value factors that incorporate differing discount rates. The spreadsheet command that will accomplish this is the SUMPRODUCT function (where the multiplication sign is used instead of a comma.) An example of this formula for the value after the first two stages is:

  SUMPRODUCT((Stage 1 Switch + Stage 2 Switch) * Period Cash Flow * PV Factor)

Computation of cash flows and values for the various stages is illustrated in the figure below (selected periods are presented to illustrated to demonstrate how the calculations change for different stages):

Period Code	-54.00	-53.00	-41.00	-40.00	-39.00	-38.00	-30.00	-29.00	0.00	1.00	2.00	3.00	4.00
Start Date	1-Apr-09	1-May-09	1-May-10	1-Jun-10	1-Jul-10	1-Aug-10	1-Apr-11	1-May-11	1-Oct-13	1-Nov-13	1-Dec-13	1-Jan-14	1-Feb-14
End Date	30-Apr-09	31-May-09	31-May-10	30-Jun-10	31-Jul-10	31-Aug-10	30-Apr-11	31-May-11	31-Oct-13	30-Nov-13	31-Dec-13	31-Jan-14	28-Feb-14
First Stage Development	6,667	6,667	6,667	6,667	-	-	-	-	-	-	-	-	-
Second Stage Development	-	-	-	-	20,000	20,000	20,000	-	-	-	-	-	-
Construction Expenditures	-	-	-	-	-	-	-	13,333	13,333	-	-	-	-
Cash Flow	-	-	-	-	-	-	-	-	-	150,000	150,000	150,000	150,000
Cash Flow	(6,667)	(6,667)	(6,667)	(6,667)	(20,000)	(20,000)	(20,000)	(13,333)	(13,333)	150,000	150,000	150,000	150,000
Period	1	2	14	15	16	17	25	26	55	56	57	58	59
Discount Rate	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.12	0.12	0.12	0.12
Discount Factor	0.99	0.99	0.91	0.91	0.90	0.89	0.85	0.84	0.69	0.57	0.57	0.56	0.56

- The final part of the analysis involves computing the expected value of the project by weighting the values by the probability of each stage. This part of the calculation is illustrated below. If the probabilities are not accounted for then the overestimation of value is dramatic as the total value is the number shown in the final stage.

Expected Value of Project	Value	Probability
Value of First Stage	(94,862)	90.00%
Value of First and Second Stage	(269,425)	8.00%
Value of First Three Stages	(575,557)	0.20%
Value of All Cash Flows	7,860,974	1.800%
Total Expected Value	33,416	100.00%

The analysis of values that include the probability of not achieving success at various stages demonstrates that an important aspect of valuation of this type of projects is accounting for probabilities of success at the different stages. While this is obvious in the above example where there is a small probability of making it to the operation phase, the probabilities are important even if there is a relatively small probability of cancellation. Using the case above, but assuming that the probability of succeeding in the first, second and third stage is 90%. 95% and 99% respectively, the value computed from considering the stages is only 83% or the value that is computed without considering the discrete values at different stages. In a case where the development is 5% of the cost of the total project, the value using the expected value calculations is 50% of the value without taking account of probability.

The drivers of value for the research option can be demonstrated by performing a scenario analysis which includes different discount rates, lengths of research periods, probabilities of success, movements of expenses to earlier or later stages, and the relative cost of development expenditures versus operating cash flow. Issues that are evaluated with the scenario analysis include:

- The scenarios that test the effects of discount rate evaluate how value increases or decreases with alternative discount rates during the research stage.
- The scenarios that test the length of the development period measure whether value is increased with the length of the option as is the case with financial options (see Appendix 1).
- The scenarios that consider alternative probability demonstrate the manner in which probabilities affect the value of the investment and how break-even probabilities can be computed.
- The scenarios that evaluate movements of expenses between stages demonstrate the importance of moving costs from earlier to later stages of the project.
- The relative effect of development expenditures and operating cash flows allows one to

quantify the relative value of the development expenditures in the overall context of a project.

Performing the scenario analysis along with a break-even analysis, a spider diagram and a tornado diagram demonstrates in consideration of the value of making development expenditures, judgment with respect to the possible range in different probabilities, expenditure amounts and phase lengths are more important than attempting to make mathematical simulations. Construction of the various different risk analysis tools is explained in Chapter 3 and is not repeated here.

The nature of the investment and its risk changes at different stages implying that the discount rates should not be the same for different project stages. After the project begins operation, the risk depends on all of the factors that cause volatility in prices, cost structure, demand and other factors. If the cash flows after operation would be determined by a fixed price contract, then the discount rate during the operation period should be relatively low. As explained in Appendix 1, the major implication of the Black-Scholes model was use of a risk free rate in evaluating options which was derived by demonstrating that an option can be created from a replicating portfolio of short-selling and buying risk free securities. While one cannot prove that the discount rate in the should be the risk free rate, it is reasonable to use a relatively low rate as the expenditures must be made and the amount does not vary with general conditions. On the other hand, if the cash flows during the operation period will be very volatile, the discount rate should be higher. During the development and construction period, the higher the discount rate, the lower the value as illustrated on the table below. If the discount rate is increased from 8% to 15%, the value increases by 22% while if the discount rate is lowered, the value declines to 86% of the base case.

	Development Discount Rate	Value of Project	Value Relative to Base
Base Case	8%	33,416	100%
Higher Development Discount Rate	10%	35,614	107%
Very High Discount Rate	15%	40,809	122%
Lower Development Discount Rate	6%	31,146	93%
Very Low Discont Rate	4%	28,798	86%

One of the features that increase the value of a financial option is the length of the period from which the option is purchased until the option is exercised. In the case of a research option in which the ultimate cash flow is the same whether the development period is long or short, the overall value of the investment decreases with longer development periods rather than increases. The longer development period means that the value of the ultimate payoff is reduced. Even though the cost of development also declines as the period is longer, this effect is offset by delaying the payoff.

	Legnth of First Stage	Value of Project	Value Relative to Base
Base Case	15	33,416	100%
Longer Development Phase	30	19,097	57%
Shorter Development Phase	5	44,516	133%

The analysis demonstrates the importance of the probability of proceeding past the development stage. Unfortunately it may be very difficult to estimate this probability, particularly if similar projects have not been completed earlier. Rather than pretending that some kind of sophisticated mathematical analysis can answer this question, it is probably more useful to evaluate how low the probability can fall before the value becomes negative. The table below illustrates that in the example used above, the probability can

fall to 8% before the value of the project becomes negative. The technique described in Chapter 3 to compute automate the calculation of break-even points can be used to compute the break-even probability given other assumptions ranging from subsequent probabilities to the costs of development and construction to the value of the ultimate payoff.

Probability	Value
15.00%	97,556
14.00%	84,728
13.00%	71,900
12.00%	59,072
11.00%	46,244
10.00%	33,416
9.00%	20,588
8.00%	7,761
7.00%	(5,067)
6.00%	(17,895)
5.00%	(30,723)

In the geothermal example, it may be possible to explore wells more slowly and separate the development into more stages. For example instead of exploring three wells in the first stage, a single well may be explored. If the first well exploration is not successful, then even if the second well may allow the project to be successful, the value would probably suggest not proceeding. Splitting up the development in this manner or pushing costs to subsequent stages may increase the overall cost of development, but it could be a valuable thing to do. The table below illustrates the effects moving costs into earlier or later stages. In the first scenario, the 100,000 of cost is moved from the second stage to the first stage. Even though the total costs of the project have not changed, the value decreases by 77,604, almost as much as the movement in cost between stages. On the other hand, if the first stage cost can be reduced to 50,000 and the second stage cost increased to 350,000, meaning that the total cost is increased by 100,000, then the value of the project still increases by 46,123. These scenarios demonstrate the importance of resolving as much uncertainty as possible with as little cost as possible early on in the project.

	Cost of First Stage	Cost of Second Stage	Value of Project	Diffence Versus Base	Value Relative to Base
Base Case	100,000	200,000	33,416		100%
Move to First Stage	200,000	100,000	-44,187	-77,604	-132%
Move to Second Stage	50,000	350,000	79,539	46,123	238%

The issue evaluated with scenario analysis is the question of how what is the value of an amount of money spent on development versus the amount of money spent on construction. This issue arises when developers charge a development fee for a project. This question also arises when a project involves multiple investors, one of which invests early in the development stage and the other invests at later stages. This issue can be evaluated by computing the relative change in value from a change in development cost versus a change in value as a function of the change in construction cost. The analysis below shows that the change in value as a function of a change in development costs is 62 times as much as the effect of a similar change in the value of construction cost relative to the overall change in value.

Development Cost	Value	Change in Value	Change in Value vs Change in Develop Cost	Construction Cost	Value	Change in Value	Change in Value vs Change in Constr Cost	Develop Cost Slope vs Constr Cost Slope
60,000	71,361			340,000	34,335			
65,000	66,618	4,743	94.9%	345,000	34,258	77	1.53%	61.97
70,000	61,875	4,743	94.9%	350,000	34,182	77	1.53%	61.97
75,000	57,132	4,743	94.9%	355,000	34,105	77	1.53%	61.97
80,000	52,389	4,743	94.9%	360,000	34,029	77	1.53%	61.97
85,000	47,646	4,743	94.9%	365,000	33,952	77	1.53%	61.97
90,000	42,903	4,743	94.9%	370,000	33,876	77	1.53%	61.97
95,000	38,159	4,743	94.9%	375,000	33,799	77	1.53%	61.97
100,000	33,416	4,743	94.9%	380,000	33,722	77	1.53%	61.97
105,000	28,673	4,743	94.9%	385,000	33,646	77	1.53%	61.97
110,000	23,930	4,743	94.9%	390,000	33,569	77	1.53%	61.97
115,000	19,187	4,743	94.9%	395,000	33,493	77	1.53%	61.97
120,000	14,444	4,743	94.9%	400,000	33,416	77	1.53%	61.97
125,000	9,701	4,743	94.9%	405,000	33,340	77	1.53%	61.97
130,000	4,958	4,743	94.9%	410,000	33,263	77	1.53%	61.97
135,000	215	4,743	94.9%	415,000	33,187	77	1.53%	61.97
140,000	(4,529)	4,743	94.9%	420,000	33,110	77	1.53%	61.97
145,000	(9,272)	4,743	94.9%	425,000	33,034	77	1.53%	61.97
150,000	(14,015)	4,743	94.9%	430,000	32,957	77	1.53%	61.97
155,000	(18,758)	4,743	94.9%	435,000	32,881	77	1.53%	61.97
160,000	(23,501)	4,743	94.9%	440,000	32,804	77	1.53%	61.97

An example of using option price models to justify high valuations was the rise and fall of valuations for companies in the biotech industry. Option valuation models were applied when the value of biotech companies was increasing and these techniques seemed to make sense because the development of drugs has a payoff pattern equivalent to a series of options to cancel the drugs. When a new drug is being developed, there are a series of stages during which the development can be cancelled, ranging from the initial scientific research, to testing on animals and finally to testing on humans. If much of the development expenditure occurs at the later stages and the probability of canceling is higher at the later stages than during the earlier stages, the distribution of cash flow is skewed with a limited downside and a large potential payoff in the upside. Given this cash flow distribution, traditional DCF cash flow techniques should not be applied, but rather option models should be used. Despite the theoretical validity of applying the option models, unrealistic parameters were used, it was very difficult to test the validity of the models and, more importantly, management often did not exercise the option to cancel development of a drug as they had a vested interest in the projects and were reluctant to pull the plug at optimal times. In this case, while the option price models certainly had merit, they were applied in an un-transparent manner and they did not reflect the true cash flow accruing to investors.

Contracting Phase Cancellation Option

When investigating a new investment that ultimately depends on cash flow from volatile prices or demands, the time period before large expenditures are made in the development phase could be considered similar to the exploration wells for the geothermal project described above. Since the option involves potentially deciding not to proceed after waiting for a period of time, the option is also analogous to the delay option discussed above. In contrast to the research phase where value is resolved by determining through internal study whether the project is feasible from a technical standpoint, information resolved during this phase is derived from external market factors driven by economic forces. To differentiate this time period from the research phase, it is labeled the contracting phase. While the contracting phase involves some expenditures, a primary feature of this part of developing projects is simply to wait for different periods before starting to construct the project.

In cases where the development is not derived from research that defines the technical viability of the project, the development expenditures can be modeled much like a typical call option described in Appendix 1. If the present value of inflows exceeds the cash outflows of at the commencement of development, then the development should be made. In most financial models, there is no provision for valuing an investment at intermediate stages of its life. Further, there is generally no provision for

modelling the potential dispersion in cash flows that drive the value of an option. If the option to cancel the investment after the contracting phase is considered as part of the overall value of the investment, then the value of the project must be computed at various stages of the investment as in the geothermal example above, rather than only at the commencement of the project. The difference between modelling the contracting phase and the research phase is that uncertainty and volatility are driven by time series analysis rather than probability of not proceeding. As with analysis of research phases, the value of the project must be computed at different stages. To investigate issues associated with valuation of the contracting option, mechanical financial modelling issues are first addressed. Next, issues in a case without mean reversion is compared with a standard call option where the value is measured for different levels of volatility, option length, interest rates and cost of development. The same issues are then evaluated with different magnitudes of mean reversion.

In evaluating the mechanics of measuring the value of an initial contracting phase in a project consider the case of a real estate development (virtually any type of project involving a high level of expenditures and volatile cash flows could be used.) Assume that the initial items in setting up the investment including feasibility studies, negotiating loan agreements, hiring contractors and other things take three years. After the contracting period, assume that the construction itself has a duration of two years. While it seems obvious that expenditures during the contracting phase should be valued using option pricing techniques, the typical financial modelling process does not account for the option to abandon the project after development. Instead, the development cost is generally simply added to the construction cost and evaluated relative to the subsequent cash inflows that are earned after construction is complete. To illustrate modelling issues associated with the value of the development option, begin with a simple project investment model without financing. In order to value the option, the financial model must incorporate the prospective value of the project at different stages of the investment as with the research option discussed above. In addition, uncertainty must be incorporated into the cash flow projections (if there is no uncertainty, then the decision at the commencement of development would not be different from the decision after development.) Thirdly, information about prospective cash flows at the time of a decision to exercise the option to cancel must be incorporated in the model. The final thing that must be included in the model is the assumption that management will make a rational decision to abandon the real estate project if the prospective value is below zero.

Mechanically, valuation of the development option can be computed by setting-up a model with a development period as well as a construction period and an operating period as described above through computing period codes and logical variables for the different stages. Once cash flows are computed (the development cash flows, the construction cash flows and the operating cash flows), the prospective value of the project at different stages can be computed. Further, a time series equation for prices, demand or other factors can be easily be computed with an index formula in an excel spreadsheet as shown below. This process is described in detail in Chapter 3 and works as long as there is no correlation between different variables, the volatility is constant and the underlying distribution comes from a normal distribution. Further, the process assumes discrete rather than continuous time as discussed in the appendix to Chapter 3. You could add other features to the equation as described in Chapter 3.

$$\text{Index}_t = \text{Index}_{t-1} \text{ x } (1+\text{volatility x NORMSINV(Rand())}+(1 - \text{Index}_{t-1}) \text{ x Mean Reversion Factor}$$

To apply the index variable, the index can simply be multiplied by the cash flow without volatility. The index begins with a value of 1.0 for the first period of the contracting phase and evolves up or down depending on the random number, the volatility and the mean reversion factor.

At the end of the contracting phase, the project value can be established from the present value of prospective cash flow. One way to compute the prospective value at this point is to assume perfect

foresight, which means value at the period after development could reflect the actual random events that result in the realized cash flow. This approach is not realistic because the perfect foresight is obviously not possible. Instead, valuation at the different stages should be derived from information available at the time. Here, the projected prices at the decision date for the entire remainder of the projection period depends on the index at value at the end of the contracting period as shown in the equation below:

$$\text{Index}_t = \text{Index}_{\text{contract period end}} + (1 - \text{Index}_{t-1}) \times \text{Mean Reversion Factor}$$

In the above equation, if there is no mean reversion, then the index for each period in the future is simply the index at end of the development period. This number can then be multiplied by each of the prospective cash flows that can then be discounted to establish the value of selling the project after development. If the prospective value is negative, then the project should be cancelled. To implement the index in the case with no mean reversion, the index at the end of contracting period is simply continued for remaining periods. This can be accomplished by using a conditional statement and setting an adjusted index equal to the previous index plus the index at the end of the contracting period. In terms of a formula, begin with a value of zero in the first period and simply add the term (period = end of contracting period) as demonstrated below:

$$\text{Applied Index}_t = \text{Applied Index}_{t-1} + \text{Applied Index}_{t-1} \times (\text{period code} = \text{end of contracting period})$$

Once the applied index is computed it can be multiplied by the cash flow which is then used to compute the prospective net present value in each period. The net present value of the project after the contracting period is used to determine whether the project should proceed. The process of computing the option value is illustrated below. Note that the volatility factor for period -2, the last period of the development, is applied to all of the other periods in the analysis:

Period Definitions									
Period Code	-4	-3	-2	-1	0	1	2	3	4
Development Switch	TRUE	TRUE	TRUE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE
Construction Switch	FALSE	FALSE	FALSE	TRUE	TRUE	FALSE	FALSE	FALSE	FALSE
Operation Switch	FALSE	FALSE	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE
Cash Flow									
Volatility Factor	1.00	1.85	2.70	3.41	1.41	1.45	1.69	2.72	1.52
Development Cash Flow	5,000	5,000	5,000	-	-	-	-	-	-
Construction Cash Flow	-	-	-	42,500	42,500	-	-	-	-
Operation Cash Flow	-	-	-	-	-	20,000	20,000	20,000	20,000
Operating Cash Flow with Volatility	-	-	-	-	-	29,062	33,739	54,497	30,384
Net Cash Flow	(5,000)	(5,000)	(5,000)	(42,500)	(42,500)	29,062	33,739	54,497	30,384
Valuation after Development									
Volatility Factor at Development		-	2.70	2.70	2.70	2.70	2.70	2.70	2.70
Cash Flow after Develpoment	-	-	-	-	-	54,097	54,097	54,097	54,097
Net Cash Flow Adjusted for Volatility	(5,000)	(5,000)	(5,000)	(42,500)	(42,500)	54,097	54,097	54,097	54,097
Prospective NPV	218,119	244,931	274,424	306,866	380,053	460,558	452,517	443,672	433,942

Since random numbers are used to compute the cash flow, multiple scenarios must be tabulated to determine the ultimate value of the option. With the random number in a formula for the volatility factor, the F9 key be pressed and the value of the option changes. If the model is structured with the periods in columns as shown above, it is difficult to simply copy the rows as was described in introducing Monte Carlo simulation in Chapter 3. Instead, one can write a simple macro to re-calculate the model over and

over again (analogous to pressing the F9 key and copying and pasting the NPV.) This can be easily accomplished with a for next loop and the CELLS command as described in Chapter 3 where time the model is re-calculated in the loop, the option value can be saved and printed into a new sheet that keeps track of different scenarios. Before running the macro, you should make sure the results a logical in a case with no volatility.

The benefit of having an option to wait in a project such as real estate development is that one has more recent information about the expectation of prices then existed at the beginning of development. If there is an evolution that increases prices and cash flow, then it is more likely that the investment should proceed. On the other hand, if the prices fall, then it is more likely that the project will be scrapped at the end of the contracting phase. The longer the development period the more information about prices and cash flow making the option more valuable. While the concepts work well without mean reversion, the presence of mean reversion dramatically affects the value of the development option as shown below. If prices come back to equilibrium levels rather than evolving from current levels, then the value of obtaining information during the development is reduced as not much more information gained during the development period because the equilibrium price is more important than the current existing price.

To model mean reversion, the applied index discussed above must be adjusted to eventually move back to 1.0. For example, if the mean reversion factor is 20% implying that prices move back to equilibrium in five years, then the index for the volatility should decline to 1.0 over about five years. This is accomplished by modifying the adjusted index through multiplying the prior index by the mean reversion factor as shown below:

$$\text{Applied Index}_t = \text{Applied Index}_{t-1} \text{ x Mean Reversion Factor}$$

The final mechanical issue in developing the model is the question of what discount rate to use and whether to use a different discount rate for the different project phases. Copeland and Antikarov demonstrate that it is inappropriate to use the same discount rate for different periods in the development of a project and that one can adjust probabilities to account for different discount rates. One could also argue that the discount rate for construction expenditures should be different from the discount rate for the remainder of the project. As discussed in Chapter 4, the discount rates. Rather than pretending that one can create discount rates, evaluate a few simpler issues associated with the option to cancel the project after the development stage.

With the model that quantifies value at different phases established, issues associated with valuing the real option to cancel can be evaluated through running a set of different scenarios. This questions include: what is the value of the contracting option relative to the value of the overall project using different volatility assumptions; what is the value of contracting expenditures relative to the value of construction expenditures; what is the effect of longer or shorter contracting periods; what is the effect of the project being deep in-the-money or out-of-the money; and, crucially, what is the effect of mean reversion on the value of the option. These issues – volatility, length of the option, exercise price and cost of the option – are directly analogous to valuation of financial options. To determine the value of the project and the value of the option, it is assumed that the project is sold at the end of the contracting period. For cases where the value of the project is negative at the end of the contracting period, the option is assumed to be exercised, meaning that the project is cancelled and does not generate any cash flow. The value of the option is the average value of exercising the option (i.e. the negative value of the project that is avoided) over multiple scenarios in Monte Carlo analysis. In summarizing the different scenarios, the value of the option relative to the value of the project in a case with no cash flow volatility is tabulated.

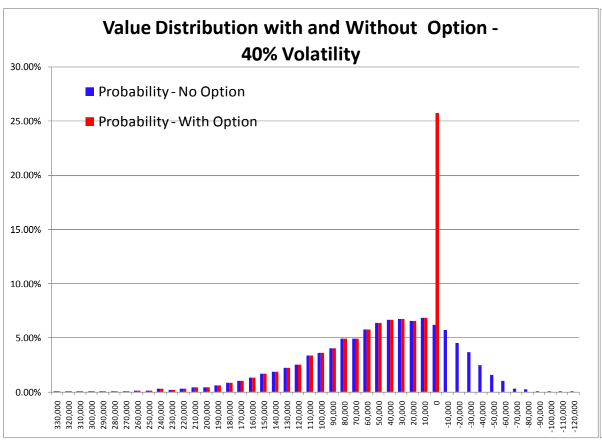
The first cases show the value of the option to cancel after the contracting phase with different levels of

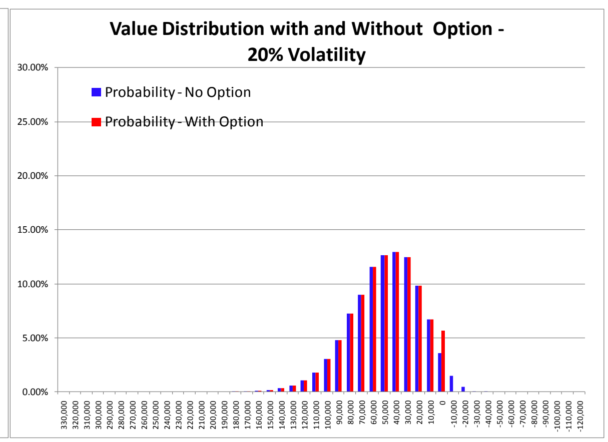
volatility and assuming that there is no mean reversion. The value of the value of the project without volatility; the value of the option to cancel and the value of the option relative to the value of the project with no volatility are shown for cases with 20%, 40%, and 60% volatility. Cash flow volatility parameters of 40% or 60% are very high and not realistic for most projects (even the volatility of oil prices in the period when price increased to $147 and then decreased to $40 the volatility was around 40%). Rather than representing realistic values, these volatility parameters are used to demonstrate how the value of the option changes as a function of different levels of volatility. The table below demonstrates that the value of the option increases with higher levels of volatility. With 20% volatility, the value of the option relative to the value of the project with no volatility is 1.64% while the value increases to 21.43% if the volatility is 40% and the value is 54% if the volatility is 60% (40% and 60% are very high volatility parameters and imply that the cash flow has a good change of falling to zero or obtaining high levels.)

Value of Option with Different Volatility

Volatility	Mean Reversion	Develop Period	Value w/o Vol	Option Value	Option to No Vol Case
20.0%	0.0%	3	31,354	515	1.64%
40.0%	0.0%	3	31,354	6,720	21.43%
60.0%	0.0%	3	31,354	16,927	53.99%

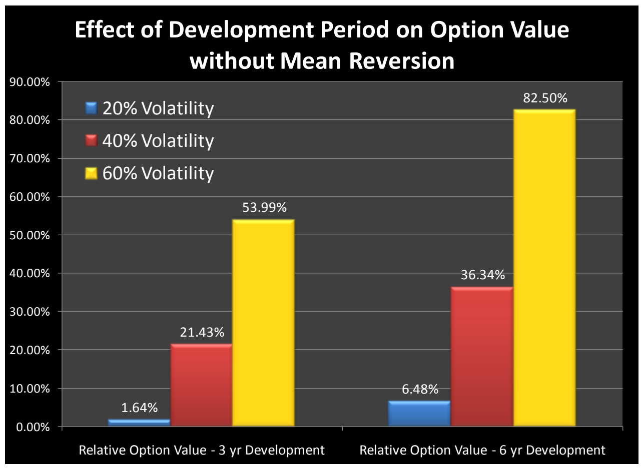
The two graphs below demonstrate the distribution of value with and without the option in the 20% volatility case and the 40% volatility case. The graph on the right shows how the option to cancel limits the possibility of negative values and that a large percentage of simulations where the option to cancel is included result in a zero value. In contrast, for the case with 20% volatility there is less distribution in the values which means there is less chance the value will be less than zero and less of a benefit from limiting the downside by canceling the project.





The second set of cases evaluates how the option value changes as a function of different lengths of the contracting period. With longer contracting periods, there is an increased chance of achieving a higher or lower value as the effects of volatility of the cash flow imply there is more of distribution in the value of the project. With more of a chance that value can below zero, the option value should increase with longer contracting periods. Counterbalancing this increased chance for exercising the option to cancel is that the fact that the longer one must wait, the higher the present value of the costs incurred for contracting and

the loss in of present value from waiting to realize the positive cash flow. To illustrate the effect of a longer contract period, the length of the period is extended to six years from the three years used in the above able. The graph below shows that when volatility is high, the value of waiting longer increases the value of the contracting option by about 30% while if the volatility is low, the increase in value is less than 5%. The effect of longer waiting periods is directly analogous to longer periods until expiration in the valuation of financial options which have the well known result that the longer the period until expiration, the higher the value of the option.



Another analogy between financial options on stocks and the real option to cancel a project after a development or contracting period is the issue of how the value of the option changes when the exercise price changes. In the case of options on stocks, if the option is in the money, meaning that the exercise price is lower than the current share price, then the option value is more than the value if the exercise price is far out of the money where the exercise price is way above the current price. For the real option to cancel after a development or contracting period, if the present value of the project before the contracting stage is expected to be very high, then the contracting option is similar to the out of the money case, as the value of the project has to change by a lot before the value become negative and the option to cancel is exercised. On the other hand, if the value of the project is marginal meaning that it is barley positive to start with, the situation is analogous to the in the money option and there is a much higher chance that the option will be exercised. To illustrate the effect of different base case cash flow levels which are analogous to being out-of-the money, the base case cash flow (the level that is multiplied by the volatility index) is increased and decreased from 20,000 to 15,000 making the value marginal. The effects of being out-of-the money are illustrated by increasing the cash flow from 20,000 to 25,000. The effect of changes in the base case cash flow using alternative volatility assumptions is shown in the table below. With the lower cash flow, the value of the project with no volatility is 4,923 which is marginal compared to the initial case and much lower than the out of the money case. In terms of absolute value of the option, the value is increased on a relative basis for the low volatility case – the value increases from 515 to 2,503. In terms of value relative to the no volatility case, the option value relative to the value of the overall project becomes much more significant when the project has marginal value. The analysis has implications in terms of the choice of different development projects, conventional wisdom is to choose safe projects with low volatility that have the highest chance of success. When taking account of the real option to cancel, the projects with higher volatility have more value and it may be more beneficial to develop a project with marginal expected value and high volatility than with a higher expected value and low volatility.

Value of Cancellation Option with Different Base Case Value					
Case	Volatility	Develop Period	Value w/o Vol	**Option Value**	**Option to No Vol Case**
Initial Case	20.0%	3	31,354	**515**	**1.64%**
Initial Case	40.0%	3	31,354	**6,720**	**21.43%**
Initial Case	60.0%	3	31,354	**16,927**	**53.99%**
In-the-Money	20.0%	3	4,923	**2,503**	**50.85%**
In-the-Money	40.0%	3	4,923	**10,091**	**204.98%**
In-the-Money	60.0%	3	4,923	**19,524**	**396.62%**
Out-of-the-Money	20.0%	3	57,785	**198**	**0.34%**
Out-of-the-Money	40.0%	3	57,785	**5,343**	**9.25%**
Out-of-the-Money	60.0%	3	57,785	**15,755**	**27.26%**

The above discussion of the real option to cancel after the development or contracting period seems to imply that keeping a lot of projects in the development pipeline is a good strategy. When the value of the option is above the present value of the development cost, making the development is worthwhile even if the expected value of the project is nothing at all. This suggests that all one has to do is spend money for contracting and feasibility and one can create value. If the value of the project is marginal, one does not have to worry as long as the market for product has volatile prices and/or demand; one simply needs to wait until the market turns around. All this appears too good to be true in the context of most investments and is an odd result given the way real world decisions are made (with notable exceptions like venture capital.) A big problem with the above analysis is that all of the above analysis ignored the potential for mean reversion in cash flow. Introduction of mean reversion has a large effect on all of the results regarding the value of the contracting option analysis and it greatly diminishes the value of the real option to cancel. Even in cases where the value is marginal and the volatility is high, the presence of small levels of mean reversion dramatically cut the option value. The table below uses the cases with volatility of 60% or 40% along with the relatively low cash flows. For these volatility scenarios, a range of mean reversion factors are applied beginning with 30% (meaning that prices take about three years to reach equilibrium), 10% (about ten years to reach equilibrium) and 5% (about 20 years to reach equilibrium). In the case where responses of firms to realized returns prompts competitive response leading to mean reversion of 30%, even in the case with very high volatility of 60%, the value of the cancellation virtually disappears. In the case with a very long mean reversion that takes a decade to reach equilibrium, the value of the option declines from 19,524 to 3,738. If the volatility is 40%, then the cancel option falls by a factor of 10 times from 10,091 to 1,074. Even in cases with extremely long mean reversion of twenty years, the value of the cancel option declines by more than half.

Value of Cancel Option					
Volatility	Mean Reversion	Develop Period	Value w/o Vol	Option Value	Option to No Vol Case
60.0%	0.0%	3	4,923	19,524	396.62%
60.0%	30.0%	3	4,923	6	0.13%
60.0%	10.0%	3	4,923	3,738	75.93%
60.0%	5.0%	3	4,923	8,939	181.59%
40.0%	0.0%	3	4,923	10,091	204.98%
40.0%	30.0%	3	4,923	-	0.00%
40.0%	10.0%	3	4,923	1,074	21.82%
40.0%	5.0%	3	4,923	4,140	84.11%

Academics and proponents of option value analysis often gloss over the issue of mean reversion. They point to the theory developed by Paul Samuelson which demonstrates the value of an investment should follows a random walk process even if the underlying prices, costs or demand are characterized by mean reversion. This is because since asset value anticipates future cash flows, the value should incorporate movements towards the mean and the remaining fluctuations in price will only be due to purely random events. Further the volatility in the asset values will be less than the volatility of the underlying movements in price and cost. A similar theory applies to the volatility and mean reversion in futures prices relative to spot prices -- forward prices should in theory follow a random walk even if spot prices exhibit mean reversion because forward prices anticipate mean reversion. There are a number of problems in applying these ideas to real options analysis. These problems include: (1) volatility of project value cannot be obtained while volatility of the revenue and cost drivers are normally the main factors considered in constructing valuation analysis; (2) the exercise of most real options such as the option do not operate when prices fall below variable cost is driven by changes in price and cost drivers; (3) mean reversion is fundamental to the economics of investments as competition drives returns to the cost of capital; (4) analysis of prices, costs and demand can incorporate equilibrium values while preparing analogous analysis for asset values is difficult; (5) while the theory is correct, stock prices of commodity firms tend be mean reverting (recall the graph of Devon Energy in Chapter 4), and (6) measuring mean reversion from historic data is very difficult. When real options are valued using economic factors that drive value such as prices and costs, it is reasonable to expect the presence of mean reversion and the mean reversion completely changes the way in which options should be considered.

Options to Delay after Development

The techniques discussed above can be extended to measure the value of the option to delay a project. Recall that the option to delay was introduced above as one of the principal options that is present in virtually any investment and supposed to change the fundamental economics of making capital expenditures with implications for marginal cost. While the option to delay may be discussed in theory, demonstration of how a financial model can incorporate evaluation of different delay possibilities in practices is rarely part of the dialogue. The practical mechanics of creating a delay option and measurement of the value of delay options is presented below. Modeling of the delay options is accomplished by modifying the previous model that included the contracting option so that a project is continually delayed until some minimum predefined hurdle value is achieved. As with the contracting option, the effects of different volatility level and mean reversion on the value of the delay option are evaluated after mechanical issues of creating the model are addressed. In the same way that the contracting option seems too good to be true before mean reversion is introduced, the option to delay appears to be valuable and to change the basic economics of investment evaluation until mean reversion is included in the analysis. The analysis below demonstrates that when small levels of mean reversion are

present in cash flows, the delay option loses virtually all of its benefits and often becomes negative.

To incorporate the delay option into a financial model, the first step is to add a delay phase into the model. The difficult part of adding this delay phase into the model is that the length of the delay phase is not fixed in advance, but rather the period length depends on the prospective value of the project. The length of the delay keeps increasing until the prospective value is above the pre-defined hurdle value, after which the project construction commences. Should the project never become economic as gauged by the hurdle value, then the delay is equivalent to canceling the project after the contracting period as discussed above. Similarly, if the hurdle value is set to zero, the delay option works much like the option to cancel after the contracting phase, except that the project is continually tested until the project obtains positive value.

Mechanically, including the delay option in a model can be accomplished using the prior model defined for the contracting option and then adding algorithms that change the delay period as a function of the prospective value of the project defined for subsequent periods. The first step in doing this is to add an input that defines the hurdle before which the project is allowed to proceed. With this hurdle criteria established, one can then add a logical variable to re-define the delay period. Specifically, the initial delay period is set to zero and then tested each subsequent period until the project becomes positive and the delay test becomes false. An example the mechanics of adding flexibility to delay a project are shown in the table below. This example works as follows:

- In the first year after the development period, the value is negative 22,234 which is below the hurdle value of 35,000 meaning that the project should not proceed (note that the delay switch titled TEST PERIOD at the bottom of the table for the period is set to true.)

- Since the value of the delay switch variable is true, the number of delay periods changed from zero to one, and the process testing and incrementing the delay is repeated by testing the new prospective value relative to the hurdle value is repeated. When this occurs, a new random number is drawn and the volatility index from the earlier period is saved. In the example below, the new random variable and volatility index still results in a negative value which prompts another year of delay.

- The process of saving the previous volatility factors and re-testing the model to see if a delay is justified is continued until the value becomes positive. Each time, the previous volatility index and the present value is maintained and the delay period is increased by a year. If the project is still negative after a pre-determined period (for example ten years) then the project is canceled.

- Eventually, the project may become profitable and construction started. In the example below, a random number draw combined with a high volatility factor results in a volatility factor of 4.01 which makes the project worth constructing. With the delay option, the project is now worth 63,227. In the case with no options, the project would have been constructed immediately after development and had a value of negative 22,234. If the contracting option was used, the project would have been canceled after the development period and it would have had a value of zero.

	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3
Period Definitions														
Period Code	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3
Development Switch	TRUE	TRUE	TRUE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE
Delay Switch	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	FALSE	FALSE	FALSE	FALSE	FALSE
Construction Switch	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	TRUE	TRUE	FALSE	FALSE	FALSE
Operation Switch	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE
Cash Flow														
Volatility Factor	1.00	1.17	2.66	0.28	0.27	0.48	0.65	0.93	1.68	4.01	3.64	3.41	2.81	0.82
Development Cash Flow	5,000	5,000	5,000	-	-	-	-	-	-	-	-	-	-	-
Construction Cash Flow	-	-	-	-	-	-	-	-	-	50,000	50,000	-	-	-
Operation Cash Flow	-	-	-	-	-	-	-	-	-	-	-	20,000	20,000	20,000
Net Cash Flow	(5,000)	(5,000)	(5,000)	-	-	-	-	-	-	(50,000)	(50,000)	68,208	56,137	16,411
Net Cash Flow - No Volatility	(5,000)	(5,000)	(5,000)	-	-	-	-	-	-	(50,000)	(50,000)	20,000	20,000	20,000
Valuation after Development														
Volatility Factor at Construction Start		-	-	-	-	-	-	-	1.68	1.68	1.68	1.68	1.68	1.68
Predicted Cash Flow After Construction	-	-	-	-	-	-	-	-	-	-	-	33,522	33,522	33,522
Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14
PV Factor	0.91	0.83	0.75	0.68	0.62	0.56	0.51	0.47	0.42	0.39	0.35	0.32	0.29	0.26
Net Cash Flow Adjusted for Volatility	(5,000)	(5,000)	(5,000)	-	-	-	-	-	-	(50,000)	(50,000)	33,522	33,522	33,522
Prospective NPV Before Delay	50,793	55,338	59,470	(22,234)	(32,806)	(30,322)	(10,961)	2,071	20,873	63,227	82,504	100,029	89,347	79,637
Hurdle Value	35,000	35,000	35,000	35,000	35,000	35,000	35,000	35,000	35,000	35,000	35,000	35,000	35,000	35,000
Test Period	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
Delay	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	FALSE	FALSE	FALSE	FALSE	FALSE

Make the calculations discussed above is difficult to compute without a bit of programming because an input variable – the delay period -- must be changed depending on output variables and also because a new set of random numbers is re-computed each time the delay is evaluated. The number of delay periods must be changed as a function of repetitive calculations relating to the value of project in subsequent periods. This can be accomplished with a macro that includes a for-next loop which re-tests the delay option in subsequent periods and saves the current volatility factors that which must be used as a base for subsequent volatility factors. If the option to delay is not exercised (meaning the construction commences) because the value of the project is higher than the hurdle amount, then the process is stopped. Code that illustrates the process is included in a file that accompanies the book. The table shows how the delay option can be summarized for a single scenario. The value of the project including the option to delay is compared to the value of the project without exercising any options and is also compared to the value of the project assuming that the option to cancel after the development is exercised. The number of years of delay is shown at the bottom of the table.

Value of Delay and Contracting Option	
Value with No Volatility	$10,443
Value at End of Delay Period with Delay Option	63,227
Less: PV of Contracting Cost	12,434
Realized Value Including Delay Option	50,793
Value of Project at End of Development with No Options	(22,234)
Less: PV of Contracting Cost	12,434
Realized Value of Project without Option	(34,669)
Total Value of the Delay Option relative to No Option	85,461
Value of Project with Option to Cancel after Contracting	-
Less: PV of Contracting Cost	12,434
Realized Value of Project with Cancel Option	(12,434)
Added Value from Delay Option versus Cancel Option	63,227
Years of Delay	6

After the value of the delay option is computed for a single scenario as shown above, it can then be computed for many different scenarios using Monte Carlo simulation in the same way that simulation was used for the contracting option discussed above. This part of the process is easy to calculate by programming a simple macro – one does not have to buy Crystal Ball or At Risk; the important part is understanding the economics of time series and setting up a flexible financial model. In programming the Monte Carlo simulation, a for-next loop is created that re-computes the scenarios with different random numbers and which saves the summary results in a separate sheet for each scenario. The primary difference between this and the simulation for the cancellation option after development is that for the program to re-compute the delay option is called after each simulation. The simulation saves the value from exercising the delay option, from exercising only the cancellation option and from not exercising any option at all. In addition, the number of years of delay is recorded for each scenario in order to evaluate the probability of exercising the option.

The analysis of the value of delay using assumptions discussed above is shown in the table below for volatility assumptions of 20%, 40% and 60% along with no mean reversion and a hurdle approximately equal to the no volatility case is shown on the table below. The average value of the project assuming that no option is exercised – meaning that the value can be negative is shown after the hurdle value. In the next two columns to the right, the value of the project with the cancel option and with the option to wait until the hurdle value is met is presented. As with the total value, these numbers are the average across the total number of scenarios in the Monte Carlo simulation. Given the total value with and without different options, the additional value created by the option and the percent additional value is shown. The table shows that the delay option does add value to the project and the higher the volatility, the higher the value of the delay option. In the case with 20% volatility the delay option had a very minor effect, but in the cases with 40% volatility and 60% volatility, the option of waiting until the value increases above 35,000 to begin constructing the project creates more value for the project. Results shown in the table seem to imply that in making investment decisions, if the volatility of cash flows associated with a project is high, then management should evaluate different trigger points before which construction should proceed.

Value of Cancel Option and Delay Option

Volatility	Mean Reversion	Hurdle Value	Value of Project - No Option	Value of Project - Cancel Option	Value of Project - Delay Option	Value of Cancellation Option	Added Value of Delay Option	Cancel Option as Percent of Value	Delay Option as Percent of Value
20%	0%	35,000	41,147	41,906	41,942	760	36	1.8%	0.1%
40%	0%	35,000	40,675	48,425	53,773	7,750	5,348	19.1%	13.1%
60%	0%	35,000	39,585	57,839	64,949	18,254	7,110	46.1%	18.0%

When establishing different hurdle values before which the project is allowed to proceed with construction, there is a tradeoff between waiting to realize a large value using a relatively high hurdle rate versus the potential for foregoing projects that have positive value in hoping for even higher value. At one extreme, if the hurdle rate is set to a value of zero, the option to delay can still have positive value. This occurs because when the project has a negative value at the end of the development period, the basic cancel option after development dictates that it should be cancelled. However, by waiting instead of canceling, volatility in cash flow may result in the project eventually yielding a positive value. In zero hurdle value case, the delay option cannot hurt the value of the project. At the other extreme, consider a situation when the hurdle value is set to a very high value. Cases in which the project has a positive value at the end of the development period mean that without an option construction would commence or the project could be sold and yield positive value. Through applying the real option to delay however, there are

situations where these positive present value projects would not be selected. It is possible that subsequent volatility in cash flows results in reduced value in future years. In such cases the value would never exceed the hurdle amount even though it could have realized positive value had it begun construction in earlier periods. Here, the option to delay can result in a lower project value then would have been realized had the delay option been ignored and the option has negative value. The table below uses the case with 40% volatility above along with various different hurdle values. Here, the scenario that results in the highest value is the case with a hurdle value of 40,000 as can be seen by inspecting the column labeled delay option value.

Value of Option and Hurdle Value

Volatility	Mean Reversion	Hurdle Value	Added Value of Delay Option
40%	0%	-	1,166
40%	0%	5,000	2,123
40%	0%	10,000	3,121
40%	0%	20,000	3,918
40%	0%	30,000	5,435
40%	0%	40,000	6,275
40%	0%	50,000	6,067
40%	0%	60,000	4,682

As with analysis of the contracting option above, the discussion of the delay option has implications that seem quite odd from the perspective of an investment decision. If the analysis above is really representative of how projects should be analyzed, then decision makers should be spending time finding investments with high cash flow volatility and then analyzing the optimal hurdle amount before which the investment should proceed. As with the cancel option, this conclusion completely changes when mean reversion is present in the cash flow. The table below shows that the presence of mean reversion can reverse the value of the delay option. In this table, the scenarios use the hurdle value of 35,000 along with volatility of 40% and 60% together with different mean reversion parameters. With 40% volatility, the presence of even a small mean reversion statistic of 10% makes the value of the delay option become negative along with virtually eliminating the benefit of the cancel option. As pointed out above, the negative option value means that the hurdle value is too high for this scenario. If the cash flow volatility is the very high level of 60%, then the value also is strongly affected by mean reversion. With a 30% mean reversion parameter, the delay option becomes negative. In cases with smaller mean reversion parameters of 10% or 5%, the value of the delay option although the option still has a positive value.

Value of Cancel Option and Delay Option with Mean Reversion

Volatility	Mean Reversion	Hurdle Value	Value of Project - No Option	Value of Project - Cancel Option	Value of Project - Delay Option	Value of Cancellation Option	Added Value of Delay Option	Cancel Option as Percent of Value	Delay Option as Percent of Value
40%	0%	35,000	40,675	48,425	53,773	7,750	5,348	19.1%	13.1%
40%	10%	35,000	40,552	40,649	39,427	97	(1,222)	0.2%	-3.0%
60%	0%	35,000	39,585	57,839	64,949	18,254	7,110	46.1%	18.0%
60%	30%	35,000	40,658	40,658	34,696	-	(5,962)	0.0%	-14.7%
60%	10%	35,000	41,162	42,372	45,336	1,210	2,964	2.9%	7.2%
60%	5%	35,000	40,429	46,290	52,293	5,861	6,003	14.5%	14.8%

If a project has a positive value which justifies construction, it is just at this time when competing companies will attempt to enter the business. More competitors will reduce prospective cash flow and

imply that the positive net present value that is generated with current cash flow cannot be maintained. This simply idea implies that cash flow will have mean reversion after high values are realized. More importantly, the competitive behavior conflicts with the general strategy of delay as when the present value is positive it suggests that value should be pounced on as positive present value opportunities can quickly disappear.

Options to Cancel During Construction

One of the real options often discussed for capital intensive assets with a long construction lead time is the option to cancel the project after the construction has started. For a project such as a nuclear plant, the Boeing 787 or a new theme park, many things can completely alter the economics of the investment over the course of the lengthy construction period. Estimates of construction costs can increase as more experience is gained from construction; demand forecasts can be revised downward if there is an unexpected recession; prices can fall because of a glut of capacity. The option to cancel a project after commencement of construction can limit the downside exposure of these factors through allowing management to cut their losses if they are brave enough to scrap a project. Of course most managers will instead convince themselves that the world will turn around and the project will ultimately be profitable rather than cutting their losses and admitting they are wrong.

The option to cancel is similar to the contracting phase option and the delay option discussed above, except that the option is not evaluated only in one period. To include this option to cancel the project during the construction period, a model must keep track of whether the project has already been cancelled because it can obviously not be cancelled more than once (the option to cancel is like an American option for financial options where the option does not need to be exercised at a single period in time.) While the option to cancel may seem quite valuable at first blush, the option becomes less and less valuable as construction progresses because prospective cash flow must be assessed against less and less incremental capital expenditures. In the first construction period, the cash flow is evaluated against the total of the remaining construction expenditures, implying that if net present value of the project was marginal in the first place, there is a reasonable chance that the project can have a negative value that justifies the cancellation. With more and more money spent on construction over time, the chances of realizing prospective negative value are reduced simply because there is less prospective negative cash flow for construction. Ultimately, at the end of the construction phase, the cancel option is the same as canceling a project after it has been completed. The only way this option makes sense if there is no chance that project will cover its variable and fixed costs when it operates. Furthermore, canceling a project at the commencement of operation means that potentially valuable options to temporarily shut it down when price falls below variable cost cannot be realized and also that the option to expand cannot be employed (these are discussed below). Further, one cannot obviously exercise the option to cancel if the project has already been canceled by virtue of exercising the option to delay indefinitely or the option to cancel after development has been exercised. This means that when some writers suggest that one can simply add up a series of different options, they are wrong. When evaluating the various options, the value of the various real options are not mutually exclusive and the options cannot be computed in separate models and then added together.

As with the delay option and the contracting option discussed above, the most difficult part of assessing the value of the cancellation option is not making a time series model as discussed in Chapter 3, but adjusting the financial model so that it can be flexible enough to assess the prospective value of the project at different dates. Specific mechanical issues associated with the including the construction option are similar to the contracting and delay option described above with the exception that the model should be flexible enough to handle cancellation at multiple different time periods. To make sure that the cancellation option has not already been exercised, a logical variable named something like “project

cancelled" can be created that is false until the project is cancelled. If the variable has a value of true, then the exercise to cancel cannot be exercised. On other hand, if the value is false, then the option to cancel is exercised if the prospective value of the project has become negative because volatility has driven down the prospective cash flow. For example, if the option to cancel after development is not exercised and the option to delay is not pending, then it is possible that the prospective value becomes negative if a random event occurs to cause the prospective cash flow to fall below zero. In this case, the value of the cancellation option is the benefit from not being stuck with the negative value.

The biggest difficulty in evaluating the cancellation option is the mechanical issue of re-computing the prospective net present value with new volatility factors each period. Recall from discussion of the cancel option above that volatility factors were set at end of the development period and then held constant (with adjustment for mean reversion) over the remaining life of the period. This establishes the value of the project at a particular date without assuming that those buyers and sellers of assets who determine the value have knowledge of future random events. In the case where the cancel option is only evaluated during one period, the established volatility factor is held constant by using a formula as illustrated below where the current period volatility is computed from a random process (with mean reversion, price boundaries and other parameters) and the end of development test is a logical TRUE/FALSE variable that is true only at the end of the development period.

$$\text{Volatility Factor}_t = \text{Volatility Factor}_{t-1} + (\text{End of Development Test}) * \text{Current Period Volatility}$$

In the above formula, the current period volatility shown on the right hand side of the equation is computed each period as random events in the world change the value of the project. The volatility factor applied to cash flows begins with a value of zero and remains there until the end of the development period, at which time it is set to the current volatility generated from the random process. After the end of the development period, the right hand term of the equation is zero again, meaning the volatility factor is always the same as the amount at the end of the development period. In this way the value is established at the current state of the world without assuming that investors somehow have knowledge of future random events.

To assess the cancellation option, the same process must be applied to each year after the construction commences to evaluate the possibility of prospective negative cash flow. To simulate a series of prospective values, each of which does not assume that investors have knowledge of future uncertainty, a matrix can be established that remembers each applied volatility factor. For example, the volatility factor from first period of construction will be constant after that first period as the second volatility factor will be constant after the second period and so forth. To program the ability to make repetitive prospective valuations with constant volatility factors one can create a matrix that keeps track of the factors for each vintage as shown in the table below. This table can be accomplished using the techniques described for computing vintage depreciation in Chapter 2 where the current volatility is transposed, a logical variable that determines the start period for evaluation is computed and after the start period the prospective volatility remains constant.

Period Code			-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9
Construction Switch			TRUE	TRUE	TRUE	TRUE	TRUE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE
Operation Switch			FALSE	FALSE	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
Construction Expenditure			280	280	280	280	280	0	0	0	0	0	0	0	0	0
Operating Cash Flow			0	0	0	0	0	200	200	200	200	200	200	200	200	200
Cash Flow			-280	-280	-280	-280	-280	200	200	200	200	200	200	200	200	200
Present Value			150.71													
Volatility			1.00	1.08	0.81	0.68	0.85	0.94	1.06	0.89	0.98	0.60	0.66	0.51	0.59	0.76
Prospective Volatility																
	-4	1.00	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	-3	1.08	-	1.08	1.08	1.08	1.08	1.08	1.08	1.08	1.08	1.08	1.08	1.08	1.08	1.08
	-2	0.81	-	-	0.81	0.81	0.81	0.81	0.81	0.81	0.81	0.81	0.81	0.81	0.81	0.81
	-1	0.68	-	-	-	0.68	0.68	0.68	0.68	0.68	0.68	0.68	0.68	0.68	0.68	0.68
	0	0.85	-	-	-	-	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85
	1	0.94	-	-	-	-	-	0.94	0.94	0.94	0.94	0.94	0.94	0.94	0.94	0.94
	2	1.06	-	-	-	-	-	-	1.06	1.06	1.06	1.06	1.06	1.06	1.06	1.06
	3	0.89	-	-	-	-	-	-	-	0.89	0.89	0.89	0.89	0.89	0.89	0.89
	4	0.98	-	-	-	-	-	-	-	-	0.98	0.98	0.98	0.98	0.98	0.98
	5	0.60	-	-	-	-	-	-	-	-	-	0.60	0.60	0.60	0.60	0.60
	6	0.66	-	-	-	-	-	-	-	-	-	-	0.66	0.66	0.66	0.66
	7	0.51	-	-	-	-	-	-	-	-	-	-	-	0.51	0.51	0.51
	8	0.59	-	-	-	-	-	-	-	-	-	-	-	-	0.59	0.59
	9	0.76	-	-	-	-	-	-	-	-	-	-	-	-	-	0.76

Incorporating mean reversion into the prospective volatility becomes a little more complex as the factor for each period begins with the random current volatility that changes over time and then once the start is established, it gradually converges to the base value of 1.0. The manner in which mean reversion changes the prospective volatility is demonstrated by the formula below which is an extension to the right hand side of the above formula:

$$\text{Volatility Factor}_t = \text{Volatility Factor}_{t-1} + (1 - \text{Volatility Factor}_{t-1}) * \text{Mean Reversion Factor}$$

To model mean reversion using a similar matrix as above, it is often effective to create a range name and then insert formulas into the range names so as the make the formula more manageable. For example, a variable named START can be defined (using the CNTL, F3 keys in excel) which is a TRUE/FALSE logical variable that is equal to TRUE when the period code on the row equals the period code on the column. Another variable named ALIVE can be created that is true after the start period so that the mean reversion process begins at the right time. The table below demonstrates the results of this process using a single simulation with a mean reversion factor of 20%. By looking at the table, you should see how the initial volatility factor begins at he current volatility as above, but then if it is above 1.0, it gradually declines to 1.0 while if it is above 1.0 it gradually increases to 1.0.

1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.15	-	1.15	1.12	1.10	1.08	1.06	1.05	1.04	1.03	1.03	1.02	1.02	1.01	1.01	1.01	1.01	1.01
0.50	-	-	0.50	0.60	0.68	0.75	0.80	0.84	0.87	0.90	0.92	0.93	0.95	0.96	0.97	0.97	0.98
0.29	-	-	-	0.29	0.43	0.55	0.64	0.71	0.77	0.81	0.85	0.88	0.90	0.92	0.94	0.95	0.96
0.32	-	-	-	-	0.32	0.46	0.56	0.65	0.72	0.78	0.82	0.86	0.89	0.91	0.93	0.94	0.95
0.56	-	-	-	-	-	0.56	0.65	0.72	0.77	0.82	0.86	0.88	0.91	0.93	0.94	0.95	0.96
1.03	-	-	-	-	-	-	1.03	1.02	1.02	1.01	1.01	1.01	1.01	1.01	1.00	1.00	1.00
1.18	-	-	-	-	-	-	-	1.18	1.15	1.12	1.09	1.07	1.06	1.05	1.04	1.03	1.02
1.66	-	-	-	-	-	-	-	-	1.66	1.53	1.42	1.34	1.27	1.22	1.17	1.14	1.11
1.28	-	-	-	-	-	-	-	-	-	1.28	1.23	1.18	1.15	1.12	1.09	1.07	1.06
0.83	-	-	-	-	-	-	-	-	-	-	0.83	0.87	0.89	0.92	0.93	0.95	0.96
0.83	-	-	-	-	-	-	-	-	-	-	-	0.83	0.86	0.89	0.91	0.93	0.94
1.15	-	-	-	-	-	-	-	-	-	-	-	-	1.15	1.12	1.09	1.07	1.06
1.49	-	-	-	-	-	-	-	-	-	-	-	-	-	1.49	1.39	1.31	1.25
1.86	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1.86	1.68	1.55

Once the volatility factor with mean reversion is established, the final step re-computing prospective value of the project in each period is multiplying the expected base case level of cash flow by the volatility factor and adjusting the value of the cash flow so that it only includes the prospective capital expenditures that have not already been spent. For example, if the remaining construction period is four years, and the prospective value is being calculated for the second period in the above table, then the base period cash flow -- say 20,000 -- would be multiplied by 1.08 meaning the value would 21,600. In the second year of operation, the mean reversion factor means that the volatility factor declines to 1.06 and the future cash flow used in computing the present value of cash flows is 21,200. Continuing with the example, assuming that one year of construction has already been spent, the prospective cash flow will not consider the first year of construction expenditures. After the future cash flows are established, the cash flows are multiplied by present value factors which reflect the time value of money in the initial period and the prospective value in each period is finally computed. These can be tabulated on the left hand side of the matrix as illustrated below (this table has no mean reversion and 50% volatility).

Period Code			-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8
Construction Switch			TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE
Operation Switch			FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
Construction Expenditure			200	200	200	200	200	200	200	0	0	0	0	0	0	0	0
Operating Cash Flow			0	0	0	0	0	0	0	195	195	195	195	195	195	195	195
Cash Flow			-200	-200	-200	-200	-200	-200	-200	195	195	195	195	195	195	195	195
Present Value			28.17														
Current Volatility			1.00	1.04	0.42	0.18	0.10	0.21	0.05	0.03	0.02	0.02	0.02	0.04	0.04	0.05	0.05
Period for PV			1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00	14.00	15.00
PV Factor			0.93	0.87	0.82	0.76	0.71	0.67	0.62	0.58	0.54	0.51	0.48	0.44	0.41	0.39	0.36
Cash Flow																	
	28.17	-6	(200.00)	(200.00)	(200.00)	(200.00)	(200.00)	(200.00)	(200.00)	195.00	195.00	195.00	195.00	195.00	195.00	195.00	195.00
	259.60	-5	-	(200.00)	(200.00)	(200.00)	(200.00)	(200.00)	(200.00)	202.85	202.85	202.85	202.85	202.85	202.85	202.85	202.85
	(256.12)	-4	-	-	(200.00)	(200.00)	(200.00)	(200.00)	(200.00)	81.12	81.12	81.12	81.12	81.12	81.12	81.12	81.12
	(349.32)	-3	-	-	-	(200.00)	(200.00)	(200.00)	(200.00)	35.91	35.91	35.91	35.91	35.91	35.91	35.91	35.91
	(285.95)	-2	-	-	-	-	(200.00)	(200.00)	(200.00)	20.18	20.18	20.18	20.18	20.18	20.18	20.18	20.18
	(28.92)	-1	-	-	-	-	-	(200.00)	(200.00)	40.36	40.36	40.36	40.36	40.36	40.36	40.36	40.36
	(67.61)	0	-	-	-	-	-	-	(200.00)	10.04	10.04	10.04	10.04	10.04	10.04	10.04	10.04
	31.53	1	-	-	-	-	-	-	-	5.56	5.56	5.56	5.56	5.56	5.56	5.56	5.56
	21.95	2	-	-	-	-	-	-	-	-	4.31	4.31	4.31	4.31	4.31	4.31	4.31

The final mechanical step in the process is to transpose the values along the column back to rows so that prospective cancellation can be assessed. The completed model with prospective valuations is illustrated on the figure below. In the simulation shown, the project has a positive value after the development period and is not delayed by virtue of the delay option. This means that project cancelled switch is set to false. However, after four years of construction, the value becomes negative because of random events in the world. Because of this the project is scrapped after the company has invested in the first four years of the project, this negative value is avoided. After this the switch is set to true so that value of the cancel option will not be double counted. Note that it would also be possible to delay the project instead of canceling the project, but the programming is more complex as described above. The value of the cancel option is computed as the difference between the option to cancel at the end of the contracting phase and the value of the cancel option after the contracting period.

Period Definitions												
Period Code	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2
Development Switch	TRUE	TRUE	TRUE	TRUE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE
Delay Switch	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE
Construction Switch	FALSE	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
Operation Switch	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE
Cash Flow												
Volatility Factor	1.00	0.39	0.40	0.50	0.87	1.40	1.16	(0.31)	(0.51)	(0.42)	(0.50)	(0.40)
Development Cash Flow	3,750	3,750	3,750	3,750	-	-	-	-	-	-	-	-
Construction Cash Flow	-	-	-	-	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000
Operation Cash Flow	-	-	-	-	-	-	-	-	-	-	-	-
Net Cash Flow	(3,750)	(3,750)	(3,750)	(3,750)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)
Net Cash Flow - No Volatility	(3,750)	(3,750)	(3,750)	(3,750)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)
Valuation after Development												
Volatility Factor at Construction Start		-	-	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
Predicted Cash Flow After Construction	-	-	-	-	-	-	-	-	-	-	-	-
Period	1	2	3	4	5	6	7	8	9	10	11	12
PV Factor	0.91	0.83	0.75	0.68	0.62	0.56	0.51	0.47	0.42	0.39	0.35	0.32
Net Cash Flow Adjusted for Volatility	(3,750)	(3,750)	(3,750)	(3,750)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)	(10,000)
Prospective NPV Before Delay	34,256	(12,280)	(11,467)	(3,872)	24,320	70,669	58,317	(48,361)	(59,056)	(48,084)	(50,291)	(39,462)
PV for Option Evaluation	34,256	(12,280)	(11,467)	(3,872)	24,320	70,669	58,317	(48,361)	(59,056)	(48,084)	(50,291)	(39,462)
Switch for Cancel at the End of Development	FALSE	FALSE	FALSE	FALSE	TRUE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE
Value of Cancel Option at End of Development	-	-	-	-	-	-	-	-	-	-	-	-
Cancel Switch	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	TRUE	FALSE	FALSE	FALSE	FALSE
Already Cancelled Switch	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE
Value of Cancel Option During Construction	-	-	-	-	-	-	-	48,361	-	-	-	-

The mechanical discussion of how to compute prospective value for evaluation of the cancellation is dense. It is not easy to write all of this and it is certainly not easy to understand all of the mechanics without working through an example. If you did not follow every single step in the process, the idea is that you at least see that developing the mechanics of a model that is flexible enough to realistically assess the value of real options in not a simple process. In discussing real options, it is not good enough to wave your hands and say that the cancellation option is valuable or can be added to a model without working through the process of how this value is computed.

After working through all of the mechanical issues, the value of the cancellation options using different volatility assumptions, mean reversion parameters, construction periods and cash flows can be evaluated. The value of the cancellation option depends on similar factors as the contracting and the delay option; namely the volatility, the mean reversion and the extent to which the project is in-the-money as measured by expected present value of the project. In addition, the cancel option is affected by the length of the construction period. If the construction period is only one year, then the option can only be exercised after it begins operation. On the other hand, if the project has a long construction period, the option can be exercised before too much money has been invested in the project. To demonstrate the potential value of the cancellation option, the three volatility scenarios are shown with a construction period of three years and a construction period of ten years. In testing the delay option, a hurdle value of zero is used which makes it more likely that a future negative value can be achieved. The table presents the incremental value with no options, the value with the delay and contracting option and also the value with the cancellation option.

INSERT TABLE

The table below shows the effect of the option to cancel during construction with and without mean reversion. The table shows the value of the project with not options, the value with the delay and contracting option and the value including the cancellation option. In addition to the values, the year of construction for cases in which the option to cancel is shown.

INSERT TABLE

Option to Stop Operating when Prices Fall Below Variable Cost

In making an investment in capital equipment, there is often a trade-off between fixed and variable cost. For example, a natural gas electricity plant has high variable cost while a nuclear plant has low variable costs that change with more energy production, but much higher fixed operation and maintenance costs and capital cost. The gas plant has the option to not operate when the price falls below the variable cost. This option not to operate is less valuable for the nuclear plant because it is much less likely that the price will fall below its variable cost. Similar fixed versus variable cost tradeoffs occur in many different industries ranging from mining to real estate to transport. The question addressed in this section is how much does the option to stop operating add to the value of an investment with relatively high variable costs and low fixed costs.

Value is less with correlation.

The remaining discussion of real options address options that arise after the construction of a project has been completed and operations have began. These options include the possibility of temporarly ceasing operations when prices fall below variable cost, the option to expand the size of the project, the option to retire the project. Examples of electricity plant or copper mine with different operating costs. Higher option value and more skewed cash flow distribution with higher variable costs and lower fixed cost. Demonstrate the skewed distribution with plants that have different cost structures. How changes with mean reversion. In particular, the case of electricity must differentiate between weather and other factors that have less mean reversion. Popular to suggest that capital intensive assets with long lead times have the option to cancel after construction. Ultimately any asset has the option to not operate when prices do not cover variable costs. If come near the end of construction, then all the option to cancel gives you is this. But then cannot operate if the prices increase. Show how the value of the option to cancel declines as construction progresses.

Option to Re-Finance Debt in Project Finance

In project finance, the existing debt can be re-financed meaning that the current debt is replaced with new debt that has a higher balance, a lower interest rate, a longer maturity or some other advantageous term. If the existing debt is replaced with new debt that has a higher balance, then the excess amount of money can be distributed as a dividend. With more debt, a longer term, or lower interest rates, the equity IRR of the project will improve. Re-financing of debt is analogous to a real option because re-financing of debt only occurs in the upside scenario where operating cash flow is greater than or more stable than operating cash flow that was expected in the base case. In a downside case, the equity sponsor does would not want to re-finance the existing debt because the terms of the re-financing would be worse than the existing debt terms.

To demonstrate how the potential for re-financing can affect the economics of a project, a model is developed with the capability to incorporate re-financing. With the model, the effects of re-financing in upside cases on the equity IRR and the equity NPV are demonstrated. Finally, a simulation model is presented which quantifies the value created from re-financing through measuring equity value with re-financing and without re-financing. The model roughly corresponds to a toll road project where the initial level of traffic and the ramp-up rate are uncertain. However, once traffic is established, the revenues are

predictable and stable as their growth rate typically corresponds to the overall growth rate in the economy.

In setting up the model for re-financing, four different traffic scenarios are presented with the various re-financing possibilities. The debt is sized using the base case scenario and a debt service coverage ratio (DSCR) of 1.4, where a goal seek formula is used to derive the leverage consistent with the debt. After the ramp period, it is assumed that the DSCR used to size debt (discussed in chapter 2 and chapter 4) declines from 1.4 to 1.2 as there is less uncertainty associated with the traffic. The tricky part is to develop a flexible model that will evaluate the optimal period for re-financing.

One of the inputs that is somewhat tricky in setting up a model that incorporates re-financing is the tenor of the new re-financed debt. Two possibilities for inputting the tenor of the debt include using a fixed tenor for the new debt or alternatively assuming that the length of the new debt will last until almost the end of the life of the project life as often defined by the concession period. If the first fixed tenor method is used, a problem arises if the tenor is greater than the remaining life of the project in which case the tenor must be truncated. To enable the truncation, you must first compute the remaining life of the project and then use a look-up technique to evaluate the remaining life for the year of the re-financing. For example, if the project life if 40 years, the tenor of the re-financed debt is 20 years and the re-financing year is 30, then the tenor must be adjusted to 10 years instead of 20. At year 30, the remaining life of the project is 10 years which means that one can simply look-up the remaining life for the project for the re-financing period. If the second method is used which adjusts the tenor according to the remaining life is used, then the buffer period at the end of the project must first be defined. With the buffer period, the project age at the buffer period can be found which in turn allows the term of the debt to be computed as the re-finance period minus the age at the buffer period.

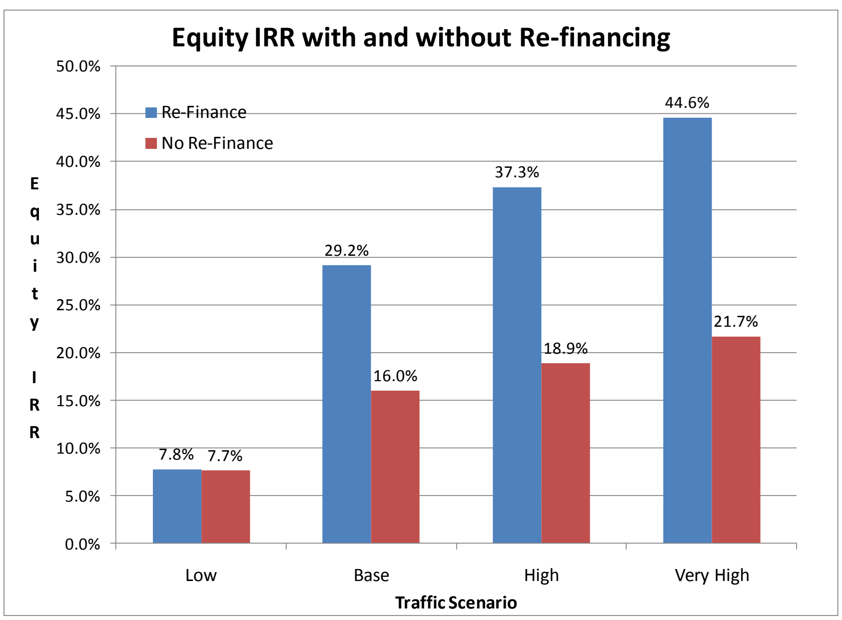
In setting up a model to incorporate re-financing, a few items that make the process work include:

- Include a calculation of the remaining life of the project
- Compute the age of the project from the re-financing date
- Make switches for the re-financing period, the pre-re-financing period and the post re-financing period.
- Construct a second sources and uses statement for the refinancing period and multiply the re-financing amount by the re-financing switch
- Add the re-financed debt in the debt schedule and compute the re-payment using the post re-financing switch
- For the initial debt, add a line for repayment at re-financing and use the MIN function with the scheduled repayment to assure that the closing balance at the re-financing period goes to zero.
- For the re-financed debt, compute the repayment using the post re-financing switch and the PPMT formula.
- In the cash flow statement include items for repayment of existing debt at re-financing and the new debt issued as well as the interest expense and repayment of debt at re-financing.
- The debt service coverage is separately computed for the pre-re-financing period and the post-re-finance period switches.

Once the model has been developed, the re-financed debt amount can be computed by using a goal seek command and finding the debt amount that results in the target DSCR. As the goal seek command is used, if you want to perform sensitivity analysis you must make a macro and then use a macro instead of a data table. In this way, you can find things like the optimal re-financing period and the effect of re-financing with different traffic scenarios.

Cash Flow										
EBITDA	-	136.50	143.33	150.49	158.02	158.02	161.18	164.40	167.69	171.04
Less: Construction	1,400.00	-	-	-	-	-	-	-	-	-
Operating Cash Flow	(1,400.00)	136.50	143.33	150.49	158.02	158.02	161.18	164.40	167.69	171.04
Add: Initial Debt Financing	777.02	-	-	-	-	-	-	-	-	-
Add: Initial Equity Financing	622.98	-	-	-	-	-	-	-	-	-
Less: Re-Financing at Maturity	-	-	-	-	-	-	555.83	-	-	-
Add: Re-Financed Debt	-	-	-	-	-	-	1,423.76	-	-	-
Cash flow Before Debt Service	-	136.50	143.33	150.49	158.02	158.02	1,029.11	164.40	167.69	171.04
Less: Interest on Initial Debt	-	54.39	52.23	49.91	47.43	44.78	41.94	-	-	-
Less: Interest on Re-financed Debt	-	-	-	-	-	-	-	85.43	84.18	82.86
Less: Repayment on Initial Debt	-	30.92	33.09	35.40	37.88	40.53	43.37	-	-	-
Less: Repayment on Re-financed Debt	-	-	-	-	-	-	-	20.78	22.02	23.34
Dividends	-	51.19	58.01	65.18	72.70	72.70	943.80	58.20	61.49	64.84
Ratios										
DSCR - Initial										
Cash Flow Beore Debt Service	-	136.50	143.33	150.49	158.02	158.02	1,029.11	164.40	167.69	171.04
Debt Service	-	85.31	85.31	85.31	85.31	85.31	85.31	-	-	-
DSCR	FALSE	1.60	1.68	1.76	1.85	1.85	12.06	FALSE	FALSE	FALSE
Minimum	1.60									
DSCR - Re-financed										
Cash Flow Before Debt Service	-	136.50	143.33	150.49	158.02	158.02	1,029.11	164.40	167.69	171.04
Debt Service	-	-	-	-	-	-	-	106.20	106.20	106.20
DSCR	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	1.55	1.58	1.61

The effects of re-financing on the equity IRR are demonstrated below. Note the in the downside case traffic scenario, the re-financing has virtually no effect on the equity IRR as the cash flow does not support higher debt capacity. However in the very high upside case, the ability to re-finance increases the equity IRR from 21.7% all the way to 44.6%. An investment with a base case return of 16% in the face of a lot of traffic risk may look very good, if the best that can be obtained is 22%. However, if the upside is 44% and the downside is only 7.8%, the investment seems a lot better.



Options from Small versus Large Investments

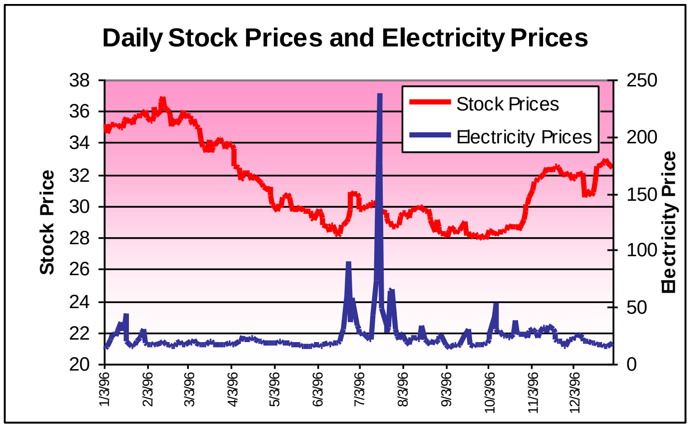
The unexpected events that occurred during the financial crisis were not all negative involving dramatically reduced stock prices, high unemployment, slowing economic growth, bankruptcies and so forth. One of the things that was most surprising was the rapid increase in the stock market that occurred after March 2009. This increase was in part due to the ability of many firms to control expenses and capital expenditures in the face of declining demand. The flexibility to cut costs and change investment strategy as demand changes can be termed a real option.

Economic Characteristics of the Underlying Security and Mean Reversion

Options are traded on financial securities – bonds, stocks and foreign exchange – and prices of various commodities – agricultural products, metals, natural gas and oil. It is possible to store all of these things through holding them in inventory. Stocks can be held in a portfolio and -- with the exception of paying dividends – the value of equity in a company accumulates all of the money that the firm has made since it was created. Similarly, foreign currencies can be held in interest bearing accounts and exchanged for other currencies and/or used to purchase goods and services. Because equity securities store value for investors and are traded in efficient markets, the best expectation of next period's price is today's price or today's price with a trend – everything but current changes are included in the accumulated stored value. The storage means that equity securities should in theory follow a random walk or a Brownain motion process. This price process also means that options theoretically maintain their value until expiration unless dividends are paid.

Since electricity cannot be stored, its value is driven by supply and demand conditions at a particular point in time and its value does not gradually build-up or move down from current amounts that are stored in inventory. Demand fluctuates in short periods because of the steep shape of the supply curve combined with demand fluctuations drives price changes and volatility. (Chapter 6) Price changes without the ability to protect against high or low prices through storage meaning that the value of electricity at expiration of an option may have little to do with the progress in prices and value over the life of the option. Therefore, if electricity options can be exercised at any time during the term of the option, it will only be on rare occasions where it is optimal to exercise the option at expiration.[6] The difference in pricing patterns between financial securities and electricity prices is illustrated on the graph below where stock prices of an electric utility company are compared to daily power prices. Note how the stock price follows a "wandering" pattern, while the power prices quickly revert back to the average level.

[6] In technical terms, electricity prices are mean reverting which means that it would have been optimal to exercise the option when the price reached its highest level rather than when it wanders over time.



Appendix A
The Black-Scholes Options Pricing Model

In 1973, Fisher Black and Myron Scholes first published their model for valuing options. The formula allowed investors to quantify the value associated with risk on an investment by plugging a few verifiable inputs into a formula. The Black-Scholes formula has been called the most applied model in economics and compared to the mapping of DNA in genetics. The equation can be derived using an arbitrage strategy where two alternative investments yield equivalent cash flows in alternative states of the world (i.e. short selling and holding risk free securities). If two strategies yield the same cash flows, they should be discounted at the risk free rate. Although several academics and practitioners have since added various modifications, the model still remains the most widely used tool for establishing the value or the premium on an option.

The Black-Scholes model uses a few parameters including terms of the option contract, volatility and the risk free rate to develop the value of the option. Given the value of an option, the model can be used to compute the volatility implied by options prices, strike prices and the term to expiration of published data. Since the model is so often applied, it is instructive to review assumptions that form the basis of the model. When valuing options on real investments, the Black-Scholes equation and other formulas that allow you to plug in a few values and pop out an answer – called closed form models – do not make much sense for a variety of reasons. A discussion of problems in the application of the Black-Scholes equation real assets with high volatility and high mean reversion is included as Appendix 1 to this chapter.

What makes the Black-Scholes equation so attractive to economists and practitioners is that the value of a complex instrument can be defined without estimating a risk premium – no beta, no market risk premium or other factors discussed in the last chapter that were so difficult to measure. The five variables that completely define value in the Black-Scholes model include: (1) the current value of the underlying asset, (2) the exercise price of the option, (3) the risk-free interest rate, (4) time to expiration, and (5) the volatility of the asset. The first four variables are readily obtainable, with volatility being the only variable that has to be estimated in any way at all.

The Black-Scholes formula can be written as applying a probability factor to the current price of a share

and the exercise price of a share (converted to present value)[7]:

Price of Call Option =
Current Price x Probability(d_1) - PV of Exercise Price x Probability (d_2)

To think about how the equation works, first consider a few different cases where we know what the value of the option should be[8]:

- In the first case pretend the exercise price is zero. This means the option will always be exercised when it expires because the value of the share when the option expires cannot be negative. The option will have the same value as the share itself and the value of the option should be the current share price. In this case the probability of the first factor is 1.0 while the second factor is zero because the exercise price is zero.

- In a second case assume that the strike price is extremely high meaning the option will probably never be exercised and the value of the option is zero. In this case the probability of both the factors is zero.

- In a third case, pretend that the risk free rate is zero, the current price of a share is 100, the volatility is zero and the exercise price is 70. In this case with zero volatility, the value of the option should be 30 because we know we that the stock price will still be 100 at expiration and we will be able to sell the share for 70. For the zero volatility third case, the probability of both the first and the second factors are both 1.0 and the value of the option is the current price of the stock minus the exercise price (100 x 1.0 – 70 x 1.0) = 30,

- In a fourth case assume that the strike price is the same as the current price, but the risk free rate is extremely high. The present value of the exercise price is very low and the real value of the expiration is close to zero meaning that this case is equivalent to the first case with both of the probability factors being one, but the value of the risk free rate side of the equation is zero.

- In a fifth case, assume that the time to expiration is very near – pretend the option will expire in one hour. In this very near term case the value of the option is the simply the difference between the current price and the exercise price as long as the current price is above the exercise price (i.e. the option is in the money.) If the option is in the money, then the probability of both factors is 1.0. If the option is out of the money, then both factors are zero and the value of the option is zero.

- In a sixth case, assume a range of different volatility parameters. The table below illustrates that as the volatility increases, the second probability term becomes lower and lower, meaning that the value of the option is closer and closer to the original price of the stock. When the volatility is lower, the two probabilities converge to being about the same, which means that the value of the option becomes the value of the share minus the value of the exercise price (the in-the-money example above.)

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[7] Much of the technical discussion of Black-Scholes the discussion in Chapter 10 of "Options, Futures and Other Derivatives" written by John Hull.

8 To see how these cases work, you can use the Black-Scholes exercise on the accompanying CD and plug in extreme values.

		Normal(d_1)	Normal(d_2)
Volatility	1%	1.00	1.00
	10%	1.00	1.00
	20%	0.98	0.95
	30%	0.95	0.83
	40%	0.92	0.70
	50%	0.91	0.58
	60%	0.90	0.49
	70%	0.91	0.41
	80%	0.92	0.34
	90%	0.92	0.28
	100%	0.93	0.23
	200%	0.99	0.02
	300%	1.00	0.00

In developing the formula, Black, Scholes and Bob Merton came up with a way to compute the probabilities as a function of the volatility, the risk free rate, the time to expiration, the current price and the strike price. They where able to convert the terms into standard normal values by assuming the stock prices follow a Brownian Motion process (Chapter 3.) Recall that if the standard normal value is zero, than the cumulative probability is about .5, while if the standard normal value is -4 or below, the probability is essentially zero and if the standard normal value is above 4 the probability is about 1.0. More precisely, the two probabilities were computed by first computing factors called d2 and d1 which give the standard normal value. Then the probabilities are plugged into a cumulative standard normal distribution to gauge come up with the option value. The present value of the exercise price is computed at the risk free rate and d1 and d2 are computed using the formulas below:

- $$d_1 = \frac{\text{Ln (Current Price/Exercise Price)} + (\text{Risk Free Rate} + \text{Volatility}^2/2) \cdot \text{Time to Expiration}}{\text{Volatility} \cdot (\text{Time of Expiration})^{½}}$$

- $$d_2 = \frac{\text{Ln (Current Price/Exercise Price)} + (\text{Risk Price Rate} - \text{Volatility}^2/2) \cdot \text{Time to Expiration}}{\text{Volatility} \cdot (\text{Time to Expiration})^{½}}$$

- Note in the above formulas that the volatility increases the first d_1 term while it decreases the second d_2 term consistent with the final example above. The time to expiration magnifies the effect of the volatility meaning when the time is large, the effect of volatility is magnified. Finally, when the current price divided by the exercise price is positive, then the value of d_1 is positive, while if the option is out of the money, the value of Ln(Current Price/Exercise Price) is negative and the value of d_1 can be negative. If the value is a large negative number, the cumulative normal probability is zero and if the log is a large positive number, the cumulative normal distribution will be 1.0.

- The normal distribution in the above formula is the cumulative probability distribution function for d_1 and d_2 based on a normal distribution with a mean of zero and a standard deviation of 1. For example, if d_1 is zero, the cumulative probability of being less than zero is .5. If d_1 is -2.0, the cumulative probability of being less than -2.0 is .0228 which is the rule of thumb that about 95% of observations are in between 2 standard deviations of the mean (i.e. 1-(2 x .0228)^2 = 95%).

In order to compute the equation above, a number of assumptions must hold. These assumptions are reviewed below and contrasted to assumptions that are appropriate for valuation of physical investments and real options[9]:

- The price of the stock must follows a random walk process with constant volatility that does not change with different time periods. This means there is no seasonality and the fact that volatility does not decline with longer time periods implies there is no mean reversion. For many if not most real investments, there are boundaries on price, shocks in prices are followed by a process where prices move to long-run equilibrium levels and volatility changes with different levels of surplus or deficit capacity.

- For the arbitrage that is used to derive the formula, short selling must be possible, so that if there are two portfolios with equivalent cash flows, one of the portfolios can be sold short and combined with debt, while a call option is available for the long portfolio. For most real assets the idea of being able to sell is in the realm of possibility. Assets can be sold, one cannot go to a traded market and create some sort of synthetic security for a toll road, a property development or most other assets.

- For arbitrage to work, there should be are no transactions costs or taxes so that portfolios can continually be created with different weights for shorted securities and bonds without any cost for the continual hedging. In the case of real assets it is not realistically possible to create some kind of real or synthetic security that will continually be re-balanced.

- The formula works for a European option in which it can only be exercised on a particular date and for the formula to work, the stock price cannot be reduced because of the issuance of dividends. In terms of real assets, this would imply that investments would be continually made in similar assets of and that no distributions to either equity holders or to lenders would occur until the option could be exercised.

- The risk-free rate of interest is constant and the same for all maturities. Over the long lifetime of an investment the cash flows arrive in many periods and the interest rates continually change.

Appendix 2 describes why a risk free rate is used in the Black-Scholes model which was the centerpiece of its development. In addition, the appendix demonstrates that the Black-Scholes model yields expected results in extreme cases with very low volatility and very low exercise prices.

Monte Carlo Simulation and the Black-Scholes Formula in Computing Different Options

Even though the assumptions required for the Black-Scholes model to derive the value of an asset do not hold for valuation of real assets (or, for that matter in financial assets), it is instructive to work through the model and use the model to benchmark results of Monte Carlo simulation. This section works through application of the Black-Scholes model and Monte Carlo simulation in valuing a both a financial option and a real option to delay construction of an investment. The exercise shows that Monte Carlo simulation can be created so that it yields the same option value as the Black-Scholes formula. This means that time series equations which include mean reversion, boundaries, correlations and so forth can be applied in valuing options without having to create a new formula every time a new factor is added to the equation.

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9 These assumptions are described in Chapter 10 of "Options, Futures and Other Derivatives" by John C. Hull. Prentice Hall, N.J., 1997.

Before valuing an option contract on a real option in which cash flows may have high volatility, mean reversion and strike prices that are not clearly defined, different option valuation models are benchmarked to a simpler case in which a financial security that follows a Brownian Motion process (Chapter 3.) In the example below, option valuation computed from the Black model (a variant of Black-Scholes) is compared to option value generated from Monte Carlo simulation. After verifying that Monte Carlo simulation produces similar results to the Black-Scholes model, then Monte Carlo simulation can be applied to more complex time series models and the comparison can be repeated with time series parameters that represent cash flows from real assets.

To compute option values using different models, consider a call option on a futures contract with an assumed 20% volatility, a current spot price of \$35 and a strike price of \$40. Assume that the contract is a European option with a one year term to expiration and a risk free rate of 5%. Unlike a stock where the value increases with the interest rate, a forward contract or a future has a value that is already defined by the discounted current spot price. To value an option on a futures contract, the Black Model, which is slightly different than the Black-Scholes model is appropriate. For a stock, if the strike price is zero, then the value of the option is the same as the value of the stock for any interest rate. This is logical because the option can be purchased for the share price and then held until expiration. At that time the option will be worth the same amount as the stock because it will be exercised. This provides the same value as owning the share itself and the value should be the same. In the case of a future, the value of the option declines with the risk free rate because if the forward contract is purchased, no money is currently invested until the contract expires. This means that the value of the option is the current spot price reduced for the cost of borrowing at the risk free rate.

The only difference between the Black model which values an option on a futures contract and the Black-Scholes Model is in the d_1 and d_2 terms. The difference involves the manner in which the risk free rate is applied as illustrated below. In the Black equation, the d_1 and d_2 terms do not have the risk free rate term because the futures contract already reflects the risk free rate:

- $d_1 = \dfrac{\text{Ln (Current Price/Exercise Price)} + (\text{Volatility}^2/2) \cdot \text{Time to Expiration}}{\text{Volatility} \cdot (\text{Time of Expiration})^{½}}$

- $d_2 = \dfrac{\text{Ln (Current Price/Exercise Price)} - (\text{Volatility}^2/2) \cdot \text{Time to Expiration}}{\text{Volatility} \cdot (\text{Time to Expiration})^{½}}$

Applying the volatility of 20%, strike price of \$40, current price of \$35 and maturity of one year to the Black equation yields values for d_1 and d_2 of -.5677 and -.7677 respectively. When these two numbers are applied to the normal distribution, the value of NORM(d_1) is.28513 and the value of NORM(d_2) is .22135. In the Black model the first term in valuation of the call option is the current price multiplied by NORM(d_1) or,

Futures Price x Normal Distribution (d_1) = \$35/MWH x .28513 = 9.979

To compute the value of the option, the second term, the exercise price multiplied by NORM(d_2) is subtracted from the first term.

PV of Exercise Price x Normal Distribution (d_2) = \$40/MWH x .22135 = 8.854

Subtracting the first term from the second term and multiplying the difference by the present value factor produces a call option value of \$1.09. If the same factors were applied to the Black-Scholes equation, then the call option value is \$1.63.

The prices generated from time series equations can be modeled with Monte Carlo simulation (Chapter 3) and used to value an option. Through adding a conditional statement (an, if test) to the data generated by the simulation, the generated data can be used to compute the value of an option. With Monte Carlo simulation, potential price changes from one period to the next are modeled with a mathematical formula combined with a draw from the random number generator. Use of Monte Carlo simulation to compute the value of an option includes: (1) simulating prices moving into the future using a drift equal to the risk free rate; (2) evaluating the payoff of the option at expiration; and, (3) discounting the payoff at the risk free rate. The value of the option is then the average across all of the simulations. Monte Carlo simulation works well in valuing European options that do cannot be exercised early.[10]

The following five step process reviews how Monte Carlo simulation can be practically applied to compute the value of an option assuming that draws are made for each day of a year.

1 Use a random number generator that produces values between zero and 1.0 to filter the random draws to a normal distribution. For example, if the draw is .5, then the normal distribution is 0; if the draw is .0027, then the draw from the normal distribution is -2.0 and so forth.

2 Convert the random draw from the normal distribution into a price movement without drift using the volatility measure. Recall that the price movement applying a normal distribution was:

$$\text{Price}_t = \text{Price}_{t-1} + \text{Price}_{t-1} \text{ x Volatility x } (1/t)^{1/2} \text{ x Draw from Normal Distribution}$$

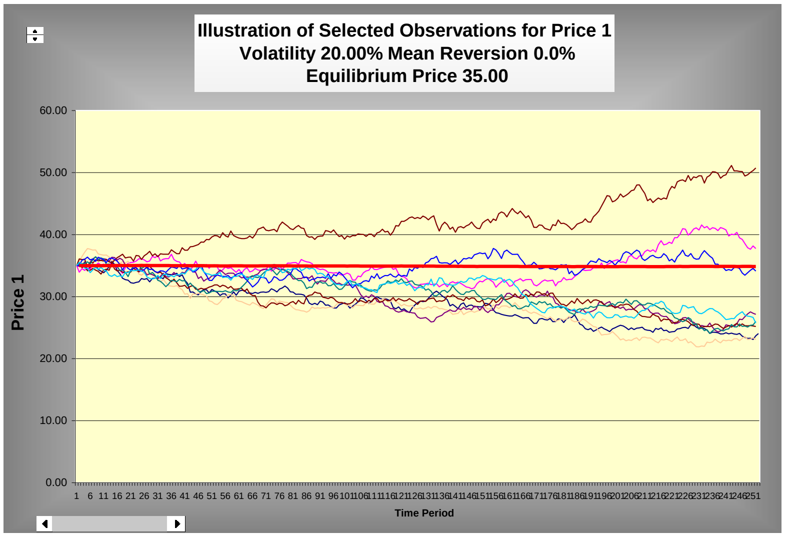
The price movement assuming a log normal distribution is:

$$\text{Price}_t = \text{Price}_{t-1} \text{ x } e^{(.5 \text{ x volatility } \wedge 2 \text{ + Draw from Normal Distribution x volatility x } T \wedge .5)}$$

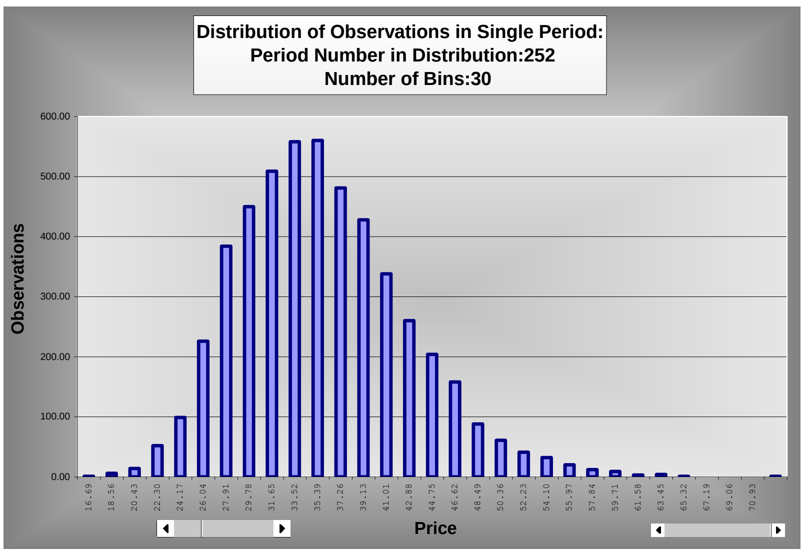
3 Complete this process of drawing from random distributions, filtering the distributions through a normal distribution and applying the standard normal value to a formula for a simulated one year time frame where the price can gradually move up or down as a function of multiple draws for each day of the simulation. For example in the first simulation, the initial $35/MWH price evolves to $35/MWH by the end of the year.

4 Repeat the process for multiple scenarios. For example, if there are 1,000 simulations and 250 trading days in a year, there are 250,000 random draws in total. A distribution of prices at the exercise date – the 250th period -- is illustrated in the accompanying graph. The second figure demonstrates simulated price movements for selected draws with relatively high ending prices and relatively low ending prices. The prices follow a wandering pattern as described in Chapter 2 associated with Brownian motion.

10 Jorion, Philippe, “Value at Risk”, McGraw-Hill, 1997, pp. 240-241.



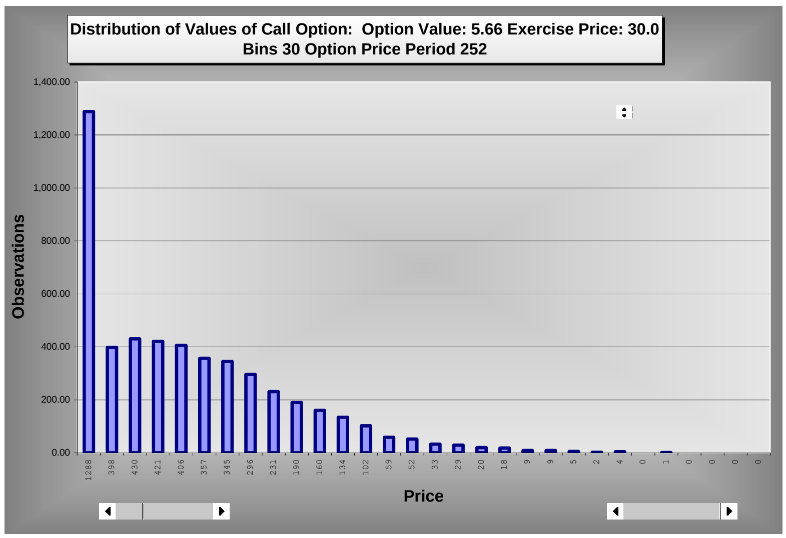
5 If a European option is simulated, sort the simulated price for the period in which the option expired. For example, if daily prices are simulated and the option period is a year, then the final price is price in period number 252 representing the days in a year. A distribution of prices for the case with 20% volatility and a time period of one year is shown on the graph below:



6 The Black-Scholes model can be replicated by using a growth rate trend in the prices which is equal to the risk free rate and using the nominal value of the exercise price or alternatively by assuming that the exercise price is discounted at the risk free rate and prices do not grow by the risk free rate. Once the exercise price is computed, then the difference between the exercise price and the final price can be computed for each scenario. For prices below the exercise price, the value of the option is zero.

7 The value of the exercise price at expiration depends on whether the option is for a futures contract or whether the option contract is for a stock or another investment in which the investment loses value over time. For an option on futures, in which money does not have to be invested, a similar process can be used. If the prices do not grow at the risk free rate, then the nominal value of the exercise can be applied. On the other hand, if prices grow at the risk free rate then the future value of the exercise price should be used.

8 To compute the value of the option using Monte Carlo simulation, calculated the average value produced for each scenario. The value of the option has a skewed distribution as illustrated in the graph below.



In applying Monte Carlo simulation to the parameters for a financial security described above with the program supplied on the CD. A Monte Carlo simulation with 1,000 different random draws is applied. In the simulation, the option is exercised in 318 out of the 1,000 the draws. The sum of the difference between the final price and the present value of the exercise price over these draws is $1,292. The valuation of the option using different time series models is illustrated in the table below:

The Monte Carlo simulation analysis does not have to use the daily prices, but could use other time increments. If larger time increments such as a year are used and fewer simulations are made, the accuracy becomes less, but the general approach still works.

The analysis demonstrates that the in the case of European option Monte Carlo simulation replicates the Black model. We now have a tool to investigate how the Black model works in alternative circumstances where there is high volatility and mean reversion. The remaining issue is whether the same replication of Monte Carlo simulation and the Black model can be applied to price series with alternative parameters such as mean reversion, lower boundaries and forward price curves.

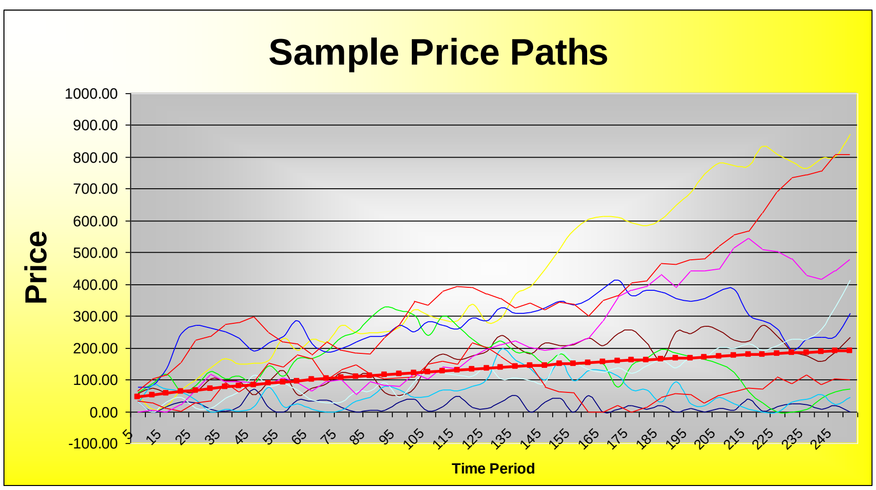
Application of the Black Model with Electricity Time Series Parameters

In the section below and subsequent sections, the Black equation and Monte Carlo simulation is used to value various call options. In the first case, a European option is modeled with high volatility and high mean reversion. To compare alternative option pricing models, volatility and mean reversion parameters for daily prices in NEPOOL are used as a case study. As noted above, the annualized volatility is 571%. Using techniques described in Chapter 2, the annual mean reversion parameter is approximately 20%.

If the Black equation is applied using a current price of $42.5/MWH, a strike price of $60/MWH and a one year expiration term results in an option value of $40.2/MWH. Deng, Johnson and Sogomonian explain that adding a mean reversion parameter does not affect the Black formula: "Note that the mean-reversion parameter of the futures price of electricity does not enter the pricing formula of an … option since the futures contracts … are traded securities and therefore the mean reverting effects are eliminated through the construction of the replicating portfolio using traded futures contracts."[11]

Application of Monte Carlo Simulation with Electricity Time Series Parameters

The analysis above demonstrated that for a financial security with a random walk process and moderate volatility, Monte Carlo simulation results in a similar option value as the Black Model. This section evaluates whether the same equivalence between the models holds true in the case with high volatility and mean reversion. Applying Monte Carlo simulation to the time series parameters also illustrates the implicit price patterns that arise from applying the parameters. As was the case for the evaluation of the Black-Scholes equation with electricity time series parameters, the Monte Carlo simulation analysis is first developed for the case with no mean reversion and then for the case with mean reversion.



11 Shijie Deng, Blake Johnoson and Aram Sogomonian, "Spark Spread Options and the Valuation of Electricity Generating Assets", Proceedings of the 32nd Hawaii International Conference on System Sciences, 1999.

The Monte Carlo technique can be used to model the pricing of options using alternative mathematical formulations. Time series equations can be developed where an assumed "reversion to the mean" and alternative seasonality parameters can be specified. As explained in Chapter 2, statistical methods for time series analysis can be used to estimate the mean reversion and seasonality parameters. Once the parameters are developed, the pricing on options can be established. The time series models can also be used to evaluate the difference between actual prices and prices computed by a simulation model. If the same daily strike process is applied to a time series with mean reversion, boundary conditions and jumps as described in Chapter 2, the value of the option is far less.

If electricity prices are applied to the simulation process the prices move to levels that are not reasonable. In applying the analysis to electricity prices, I have constructed a Brownian motion process with volatility of 517%. In one case, the prices are allowed to become negative and in a second case a lower bound.

Valuation of European Option		
	Value of Call Option To Sell Above Strike Price	Value of Put Option to Buy Below Strike Price
Total Value	2,079,958	108,992
Number of Scenarios	15,000	15,000
Value per Scenario	**138.66**	**7.27**
Risk Free Rate	0.00%	0.00%
Holding Periods	1.0	1.0
PV Factor	1.00	1.00
Present Value of Payoff	**138.66**	**7.27**

of zero is included. The Monte Carlo simulation in the case with a lower bound of zero yielded a range of prices illustrated in the graph below. Notice that the prices in many scenarios decline to zero and in some cases rose to high levels and stayed there. Clearly these prices are not plausible representations of the movement in electricity prices over the course of a year. If prices are constrained to be above zero, the option value with Monte Carlo simulation is 139. If prices are allowed to become negative, the value of the option is 87.

In the next case, 20% mean reversion is included in the analysis along with a lower bound. In one case, a volatility of 571% is used without a jump process. In a second case, a lower volatility of 300% is used along with a jump process. With mean reversion included, the value of the option is dramatically reduced. Without a jump process and with 571% volatility, the option value is 4.03. With the jump process

Real Options Effect on Value

This explains valuation of real options applied to various real options using a simplified example. The example is presented with and without mean reversion and with different levels of volatility. Use Monte Carlo simulation and include quote from Fooled by Randomness. In the cases compute the value of the asset with

the option and the value without the option. Simple example forces one to work through how to set up the model and the implicit assumptions that must be made. Exactly how the if statement works and how an assumption must be made about the future value of an asset.

In each of these cases, the flexibility management has to change decisions depending on the uncertain outcome of some variable means that cash flow upside cash flow can be increased and downside cash flow can be limited. The presumption of applying options models in valuation is that management will not make these decisions unless they increase cash flow. Each of these rights is analogous to an option on a financial asset; they are called real options. Analytical models that value derivatives can be applied to determine how each of these rights affects the value of an investment.

Valuation of Peaking Capacity and Options Pricing Models

In this section, I turn to the question of how peaking capacity can be valued using option pricing models. The analysis demonstrates that because of the extreme mean reversion in electricity, option models are not a useful tool for capacity pricing. Valuation of peaking capacity depends instead on evaluating the frequency and duration of price spikes that cause prices to spike during capacity constrained periods. Volatility around price spikes does not change valuation of pure peaking capacity relative to analytical models that use the framework described in Chapter 4.

In addition measuring a peaking plant, the exercise demonstrates a few other analysis techniques. These techniques include:

- How to compute capacity value from a spark spread on historic data.
- How to model price spikes in a time series context
- How to apply a non-normal distribution in Monte Carlo simulation

To consider valuation of peaking capacity with option pricing models, I consider three alternative valuation techniques:

- Peaking capacity valuation using historic prices without simulation
- Peaking capacity valuation using price simulation and option models
- Peaking capacity valuation using statistical analysis of historic prices without simulation

Peaking Capacity Valuation from Historic Price Data

To compute the actual value realized by peaking capacity, one can determine how many hours the price is above its variable cost and then sum the hourly margins to yield an annual capacity value. The case involves valuation of a new combustion turbine plant fired with natural gas using NEPOOL prices discussed above. In order to establish capacity value from historic data, a simple five step procedure can be applied. First, the daily prices for electricity and the operating cost of the plant are established. Second, the simulated dispatch is developed by comparing the electricity price to the variable cost of the plant. Third, the energy margins are summed for the period analyzed.

1. Establish Daily Prices and Spark Spread

Average price in NEPOOL --

Spark Spread/MWH = Electricity Price/MWH – Gas Price x HR

2. Compute the Energy Margin per MWH

In hours when the spark spread is above zero.

Energy Margin/MWH = Maximum(Spark Spread/MWH,0)

3. Sum the Energy Margin

This may be for a year or for a portion of a year or for multiple years.

4. Adjust for the Number of Hours in a Day to Develop the $/kW/Year

If the energy value is summed over a year, than the capacity value is simply the sum of the energy value per MWH for each hour the plant runs. This is simply the conversion of a megawatt hour to a megawatt year. To convert the value to a kilowatt year rather than a megawatt year, the number should be divided by 1,000. Therefore, if the summation is for a year, the formula is:

Capacity Value/kW/Year = $\sum$ (Energy Margin/MWH_i)/1,000

If prices are stated in terms of MWH but the prices are the average for 16 hours a day, the capacity value must be adjusted for the average number of hours in the period. Say the average market price per MWH for a 1 x 16 block is above the variable cost per MWH of operating a plant, the plant would be assumed dispatch for all 16 hours even if there is only one very high cost hour in the 16 hour block and the other hours are below the variable cost. In case where the analysis of 16 hour blocks is included for each hour in the year, the summation must be adjusted for the 16 hour blocks. If the number of periods for which the energy margin is computed is 52 weeks of 5x16 blocks, the energy value/MWH is multiplied by 80. The case study in the Workbook applies 16 hour blocks and uses 3.5 years of price data. Therefore, the multiplication is by 16 hours and divided by 3.5 as illustrated below:

Capacity Value/kW/Year = $\sum$ (Energy Margin/MWH_i x 16)/3,500

Applying the New England prices and natural gas prices in this framework generates an energy value of $___/kW. This is the actual value that a plant would produce. The question is whether this valuation misses something through not accounting for volatility. In the case of stock prices, the process of attempting to establish value through reviewing historic prices would understate the value of options. The historic analysis would miss the potential for prices to wander upwards because of new information.

would not work because the whole idea of an option is that new information can cause a stock price to wander. With extreme mean reversion, this does not occur.

The sum of the difference between the price and cost of operating the peaking capacity is the value of the

peaking capacity. The dollar value of peaking capacity is illustrated in the accompanying graph – the dollar value is the amount between the dispatch cost and the price level. This approach does not work with stock prices because it does not capture the fact that stock prices can wander "aimlessly" upwards or downwards to very different levels from actual prices. Therefore, in the case of prices that follow a random walk, this type of historic price analysis will not capture the potential high prices that occur when the price moves to levels that are far greater than historic prices. On the other hand, when capacity value is driven by price jumps that revert back to mean levels rather than following a non-stationary random walk, the method of using historic data does not have this basic problem.

If prices are very volatile from day to day, but have similar patterns from one year to the next, the method of arranging historic data can reasonably value peaking capacity. In this case, simulation of the fact that prices might wander is unnecessary because the prices that matter – price spikes – are unaffected by the wandering nature of earlier data. Further, if parameters – volatility, mean reversion, boundaries and jumps -- are estimated from historic data and if the market structure does not change, simulation should not change the measured value of peaking capacity.

Peaking Capacity Valuation Using Simulation and Option Price Models

In considering valuation of peaking capacity the manner in which a jump process is estimated and simulated must be determined. Recall from the discussion in Chapter 2 that the parameters involve the probability of a jump, the size of the jump, the standard deviation of the jump and the mean reversion of the jump. For the remainder of this discussion I assume that the mean reversion parameter is 1.0 implying that there is no memory in prices.

The starting point for analyzing a jump process is defining what constitutes a jump. Assuming the cost of operating a peaking plant is based on the cost of natural gas at a 11,000 heat rate. I define a jump as any occurrence above the cost of running the plant with the 11,000 heat rate. Using the SPP data, ___ jumps occurred.   The average value of the jump was $___/MWH and since 1997, the standard deviation was $___/MWH. Using the actual data, the value and summing the hours of value over the year, the value of peaking capacity was $___/kW/Year in 1997, $___/kW/Year in 1998, $____/kW/Year in 1999 and $____/kW/Year in 2000.

Using Monte Carlo simulation, there is a probability that the price will be higher than the historic prices and a probability that the prices will be below the historic levels. The simulation can capture the option value of the high prices. Note that since the mean reversion parameter is 1.0, there is no necessity to create a time series model. Contrast this with the random walk, where time series analysis is crucial to capture the effect of prices "wandering about."

A problem with this approach is that applying the Monte carol simulation with draws from a normal distribution ignores the skewed nature of price jumps. Once the skewness is accounted for, the results are similar to the analysis with historic averages.

Peaking Capacity Valuation Using Statistical Analysis of Historic Prices

The analysis can be extended in a number of different ways. Some of these include:

- Modeling price jump probability and the level of price jumps as a function of load or load less outages. Using this approach, structural shifts in capacity and load can be accounted for.

- Modeling price jumps as a spark spread or residual in regression of electricity prices versus gas

prices.

- Modeling different increments of price jumps with alternative probabilities. For example, the first increment could be between $40/MWH and $100/MWH while the second is between $101/MWH and $500/MWH and the third is above $500/MWH.

The added analytical capabilities from the simulation approach include:

- Different values can be obtained by alternative parameters for the standard deviation and probability parameters.

- Variation in loads can be modeled.

Peaking capacity is driven by the size of price jumps and the probability of the jumps occurring, not price volatility or the standard deviation of the price jumps.

Valuation of Real Options Using Long-Term Analysis: Provider of Last Resort Value

A call option has a cash flow distribution with a large upside and a limited downside. A call option also involves taking some action – exercising the option – that depends on how the ultimate level of uncertain future price levels. Many "real" assets and policy considerations have analogous characteristics where future actions can be taken depending on the level of future uncertain cash flows. Valuing assets using the notion of real options has become an increasingly popular concept. Stewart Meyers wrote a famous article in 1976 suggesting that investment analysis performed in many companies tends to understate valuations by overstating the discount rate and neglect "strategic value."[12] Meyers demonstrated that valuation of many assets with strategic elements such as research and development expenditures where management has flexibility is better analyzed using option pricing concepts than traditional discounted cash flow techniques.

Real options analysis can be used to quantify the value of a number of issues associated with electricity generation ranging from life extension of a coal plant to the level of stranded investment charges. Unlike financial options, the terms of real options are less defined in terms of the strike price and precisely how the option can be exercised. The strike price itself may be uncertain and the option may not be a decision that resolves all uncertainty. In other words, unlike a financial security where the stock can be sold and a certain profit can be realized, exercise of a real option may not guarantee realization of profits. Instead, exercise of the option could make it much more likely that an outcome will be positive, such as life extension of a power plant after prices are known to be high because of a low level of capacity expansion in a market region. Some examples of real options in the electric generation business include:

- Delaying construction of a merchant plant after permits and land has been obtained.
- Options included in purchased power contracts that increase or decrease capacity prices and cash flows depending on some trigger such as natural gas prices.
- Extending the life of power plants through making additional expenditures on environmental equipment.
- Canceling construction of a project with a long construction lead time if expected future cash flows are not sufficient to produce and adequate rate of return on the remaining non-sunk construction expenditures.

[12] See Meyers, Stewart, " ",

- Regulatory options that cause price increases or decreases depending on some future event such as provisions on a price cap that allow prices to increase.
- Retiring or continuing operation of a power plant implying that the basic expected plant life is a real option and that simply applying one estimate of a plant's life may understate its value.
- Adding the capability to switch between fuels such as natural gas and oil.

The process of valuing real options is similar to the process used to value short-term options and capacity, except that the time period is much longer. This section illustrates how real option concepts can be applied in valuation of electric generation assets used in the context of being a provider of last resort. The analysis below demonstrates that without accounting for value of providing power as a "safety net," significant policy mistakes can occur. The longer-term analysis of real options associated with being a provider of last resort raises the following issues:

- How can volatility and mean reversion be computed without historic data.
- How should forward price analysis be used in constructing time series models used to develop the option value.
- How can economic principles and judgment with respect to the nature of electricity be used in combination with historic data in developing time series parameters.
- How can supply and demand models described in earlier chapters be used to simulate volatility and mean reversion parameters for the option pricing analysis.

While there are similarities between valuing valuation of real options and financial options, there are also significant differences. Some of these differences include:

- The exercise price is not necessarily constant or perfectly certain.
- Exercise is not instantaneous – much of the information can be resolved but not necessarily absolutely all of the information.
- Even if historic data exists, changes in the structure of the market in the long-term may render the historic information useless.

Since real options are messier than financial options, use of a case study is instructive. The application I use is the situation where a utility company is the "provider of last resort" at a fixed price and retail customers can shop for power at market rates. As stated in the box above, the bankruptcy of Pacific Gas and Electric is arguably the result of customers exercising their option to secure power at a rate cap.

Case Study: Value of the Customer Option to Use Regulated Service of Market Based Pricing

In April of 1994, two American States created shock waves in the electric utility industry by announcing that they would allow their customers to directly buy generation service from competing suppliers – known in the industry as "retail wheeling." These two states were Michigan and California. While California proceeded rapidly and ultimately resulted in a financial crisis, Michigan moved at a slow pace and made the customer option to return to regulated service a central component of public policy. This notion of a customer option to return to regulated service is a real option as was the requirement of California utility companies to provide service at capped rates. Valuation of the customer option and the utility company obligation to provide regulated service as a "last resort" is central to evaluation of various public policy options and has become known as in the industry as being a provider of last resort.

In order to illustrate the valuation of a real option, I use a case study of the provider of last resort. In Michigan this option involves customers having the option to accept either market based rates or prices derived from regulated "embedded" cost of service. Since embedded costs are much more stable than

the market prices, the embedded cost can be considered the strike price. In computing how valuable this option is to customers (and how costly the obligation is to the utility company) I suggest the following step by step process can be used.

Step 1: Define the option terms – the strike price, the term of the option, and the strike provisions.

Step 2: Determine the long term time series parameters – volatility, mean reversion, and price boundaries -- using data and judgment.

Step 2: Develop a time series model from the parameters and forward prices

Step 3: Compute the exercise price on a long term basis

Step 4: Adjust the exercise method depending on the structure of the regulatory

Step 5: Run the simulation model and compute the value of the option.

In the remainder of this section, each of these steps is considered

Step 1: Define Terms of the Real Option

In order to implement competition, the State of Michigan was faced with policy consideration involving transition charges, customer Compound options and many other resource allocation decisions. Transition charges are additions to customer bills that compensate utility companies for "stranded investments" that were made before deregulation (primarily nuclear plants and uneconomic long-term contracts) and would cause financial loss to utility investors if generation prices are at market. To implement competition, the value of generating assets had to be established under the policy that was ultimately chosen. The notion of allowing customers to switch between market pricing and regulated service is established in the Michigan legislation:

"To allow and encourage the Michigan public service commission to foster competition in this state in the provision of electric supply and maintain regulation of electric supply for customers who continue to choose supply from incumbent electric utilities." [13]

This language demonstrates that the principle of customers maintaining the option to retain regulation of electric supply for customers who choose supply from incumbent electric utilities is central to the restructuring plan in Michigan. The obligation to provide service at regulated rates can be defined as a "call option" that is provided by the host utility company to Michigan customers because customers have the right -- but not the obligation -- to purchase power at market-based prices with a "strike price" equal to the generation component of bundled rates. The terms of the real option, whether there is a strike price that differs from the market price and provisions by which customers can exercise their option through switch are crucial in valuing the real option.

Step 2: Value Assets without Real Option Using Discounted Cash Flow

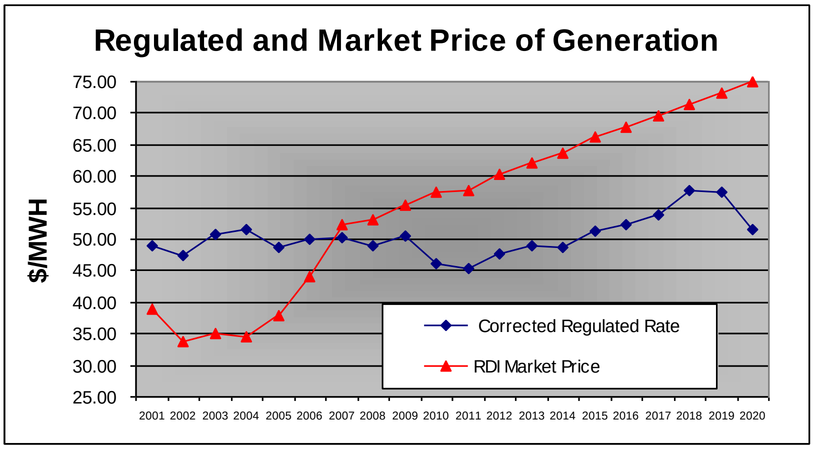
Electricity prices are very important to industrial customers such as automobile manufactures and steel companies in Michigan. Industrial groups formed an organization and argued that Michigan should not

[13] See Public Act No. 141 of 2000, Sec. 10(2) for the State of Michigan.

allow positive transition charges because of valuations derived from discounted cash flow models. These models used forward price techniques analogous to the methods described in Chapter 3 and Chapter 4 and recognized that prices may be depressed because of new capacity additions.

In applying the discounted cash flow approach to stranded investment, the industrial group computed the present value of cash flows that assets can realize at market clearing price forecasts and cost structure assumptions. The discounted as flow model values assets without accounting for the flexibility of consumers to switch between market prices and regulated rates. The basic difference between real options analysis and discounted cash flow is accounting for flexibility in reaction to volatile prices.

Step 3: Assumptions Necessary to Value Assets as Real Options



In considering the value of the customer option, one must make certain assumptions with respect to customer behavior and the exercise of the options. I initially make some simplifying assumptions and I subsequently change these assumptions. The initial assumptions include:

1) Customers have information with respect to market price levels that will be present in the next year;

2) Customer decisions to select open access service or regulated service can be gauged from aggregate cost per kWh data on the generation portion of bundled rates and aggregate market clearing prices rather than on class specific information with respect to market prices and regulated rates;

3) Market price forecasts presented in the case will turn out to be true and there is no uncertainty associated with his price forecast;

4) The regulated cost per MWH of generation is stable and can be estimated with on a reasonable basis from analysis of accounting costs; and,

5) Customers select market based pricing or regulated tariffs depending on which option results in lower electric bills without a time lag and without a "hurdle" savings criteria.

These assumptions mean that customers know what prices will be for the next year and that they can

switch back and forth between regulated rates and market prices as a function of actual market prices for the year. In fact, customers often lock into decisions for a period of one year, and perfect foresight with respect to market clearing prices does not exist. However, the "foresight" assumption is reasonable as a starting point. First, the analysis is simpler with this assumption and adding the complexity of a one-year uncertainty adds complexity to the analysis without altering the fundamental points. Second, since market prices are a function of merchant capacity, economic activity, primary fuel prices and other factors that are reasonably known ahead of time, the assumption of one year foresight may be reasonable as a starting point. Third, I will relax the assumptions and demonstrate that it does not have a significant impact on my conclusions.

Even though there is no volatility assumption in the simple case, these simple assumptions demonstrate how important consideration of real options can be in management analysis. By reflecting the expected price behavior of forward prices and regulated rates, the value of the real option is significant. Using option terminology, the option is "in the money." The value of the in the money can be measured simply by adjusting cash flows as a function of whether forward market prices are above or below regulated rates. In the years where forward prices are above regulated rates, the provider of last resort option kicks in:

For years where Forward Price < Regulated Rate

$$\boxed{\textit{Cash Flow}_t = \text{Forward Price}_t - \text{Operating Costs}_t}$$

For years where Forward Price >= Regulated Rate

$\textit{Cash Flow}_t = \text{Regulated Rate}_t - \text{Operating Costs}_t$

$\textit{Asset Value with Option} = \sum (\textit{Cash Flow}_t) \times e^{-kt}$

In this formula, k is the cost of capital to apply to free cash flow and operating costs reflect all "to-go" costs including income taxes. Incorporating the expected value of market prices and regulated rates changes the asset value for one utility company by $1.567 billion from 2.745 billion to $1.178 billion. If expected forward prices had a different pattern, or if expected forward prices were lower, or if regulated rates were higher, the value of the option being in the money is much less.

Step 4: Incorporation of a Time Series Model in the Analysis

Reflecting real options that are in-the-money can have a significant effect on investment analysis even if volatility is not computed or applied. The discussion in this chapter has established that the greater the volatility of prices, the more the value of an option. This is just as true for call options on stocks as it is for the option customers have to return to regulated generation service in Michigan. If you own a call option on a stock and if there is a great deal of variation in the price, you have more of a chance to make a lot of money because the stock price might reach very high levels. On the other hand, your downside is limited because you have a right not to exercise the option. This type of insurance exists for customers in Michigan who can select regulated service at cost based rates if market prices reach high levels. The greater the volatility of prices, the greater the value of the option to use regulated generation service because the downside risk is limited. If the value to customers is greater, the obligation that must be provided by the utility company is also greater.

To consider how volatility affects value, we must add a time series equation. At this point do not change

assumptions with respect to customer behavior – customers are assumed to be rational and have information with respect to year-ahead prices. When adding volatility to the analysis, I also incorporate other “time series” elements including price boundaries and mean reversion.

In the case of electricity, measured volatility using daily prices can be very high because of price variation driven by changing weather, price spikes, seasonal price patterns, on-peak versus off-peak prices and supply shortages. Long-term volatility is far less than the short-term volatility because over the course of a year, factors such as weather, outage-related supply shortages, and seasonal price patterns “average out.” In the long-term, volatility of electricity prices is driven by volatility in capacity prices, volatility in natural gas prices, volatility in the productivity of new technology, volatility in annual demand, and in some cases volatility in hydrology conditions. It is long-term volatility rather than short-term volatility that is required for analysis of the option value realized by customers from the option to switch between market based rates and regulated rates.

The movement of electricity prices is driven by volatility as well as mean reversion parameters and boundary conditions. There is a lower bound on electricity prices defined by short-run marginal energy costs because if prices are below marginal cost, companies will not operate plants. Mean reversion exists in electricity prices because competitive electricity prices are driven by supply and demand conditions, implying that the prices revert back to levels determined by the cost of production. An upper bound on electricity prices is also reasonable to assume because of price elasticity of demand and supply. A time series equation that includes volatility, mean reversion and boundary conditions can be represented using the following formula that we have discussed in Chapter 2:

$$
\begin{aligned}
\text{Price}_t = \text{Price}_{t-1} + {} & \\
& \text{Mean Reversion Factor x } (\text{Price}_{t-1} - \text{Forward Equilibrium Price}_t) + \\
& \text{Volatility Percent x Price}_t \text{ x Draw from Normal Distribution}
\end{aligned}
$$

[Subject to lower bound: if $\text{Price}_t < \text{Marginal Cost}_t$, $\text{Price}_t = \text{Marginal Cost}_t$]
[Subject to upper bound: if $\text{Price}_t > \text{Upper Bound}_t$, $\text{Price}_t = \text{Upper Bound}_t$]

The forward equilibrium price was estimated in this case using a supply and demand model similar to the modeling process described in Chapters 4 and 5 was applied. This process makes a number of assumptions regarding new merchant capacity, demand growth, fuel prices, plant performance and many other items. The volatility, mean reversion and price boundary conditions are a method of adding uncertainty to the forward price analysis.

Step 5: Consider Drivers of Long-Range Volatility and Mean Reversion

There is not enough historic data on competitive prices to establish volatility and mean reversion parameters for purposes of creating a time series model for electricity. Furthermore, future changes in the structure of the market imply that past data may not be appropriate for estimation of future volatility around the forward price projection even if historic data did exist. Given problems in using historic data to estimate future volatility, one must use some judgment in developing time series parameters. Because of the difficulties in developing precise time series models for the purpose of measuring option value, I use a range of volatility estimates.

Long-term volatility in electricity prices is driven by a number of supply and demand factors:

1.) There have been prolonged periods -- sometimes lasting many years -- of very low or zero capacity prices driven by slow demand increases and excess capacity. Since

capacity prices can be at levels which do not justify construction of new capacity for many years, this implies a high degree of volatility in capacity prices.

2.) Other competitive capital intensive industries with large construction projects such as mining and real estate have a high level of price volatility.

3.) There have been significant changes in the productivity of generating assets as evidenced by lower real per kW cost and improved heat rates for combined cycle plants as well as the increased capital costs of nuclear plants from the 1970's to the 1980's.

4.) Primary fuel prices for natural gas and oil significantly affect electricity prices and these commodities have high long-term volatility in the range of 20 percent to 30 percent.

5.) A large amount of year-to-year price variation can occur because of hot summers, cold winters, recessions, dry river conditions, longer than expected maintenance outages and fuel supply problems.

6.) The long life span of assets and relatively slow demand growth mean that prices move very slowly to equilibrium after a significant supply change.

By now it is clear that because electricity prices are ultimately driven by forces of supply and demand, the prices eventually move back to the long-run cost of production. In considering influences that affect price behavior over the long-run, mean reversion parameters must be considered along with volatility in developing a time series model. Some of the determinants of the level of long-term mean reversion in electricity prices include:

1.) Reversion back to the mean after price changes caused by fluctuations in weather related demand is fast, if not immediate. If prices change because of a hot summer or a cold winter, these factors should not re-cur in the next period and the mean reversion factor should be 1.0. On the other hand, demand changes due to variation in general economic conditions have impacts that last over the course of a business cycle, suggesting a slower rate of mean reversion.

2.) Reversion back to the mean price levels after an increase in new capacity construction can take a long period of time. If capacity prices are low because of surplus capacity, then plant retirements and load growth are required to move capacity prices back up to cost of production equilibrium levels. If capacity prices are high, the time lag required to permit and construct new plants drives the speed of mean reversion.

3.) The speed of mean reversion resulting from price shocks caused by changes in primary fuel prices corresponds to the speed of mean reversion in these price processes. The time series process for oil and natural gas is known to be mean reverting as exploration and production reacts to price changes in much the same way as new capacity responds to electricity prices.

4.) Price changes that arise from changes in productivity of new generating resources probably do not revert to a mean level and should follow a random walk process. Once a change in technology occurs, the cost of production changes and prices to not revert back to the prior cost of production level.

. In order to cover different possible outlooks with respect to long-term volatility in correcting Mr. Seiple's analysis, I have assumed annual electricity prices volatility of 10%, 15%, 20%, 25% and 30%.

I have made the following assumptions with respect to time series parameters other than volatility:

1.) I assume a mean reversion parameter of 25% which implies that after a "shock", electricity prices move back to within 68% of the mean "equilibrium level" within four years.
2.) I assume a lower boundary on prices of $15/MWH, which is intended to approximately reflect the levels of short-term marginal cost in ECAR.
3.) I assume an upper bound on all-hour (both on-peak and off-peak) prices of $60/MWH that accounts for construction of peak capacity and interruption in loads when prices rise to very high levels.
4.) I assume that the lower and upper bound prices escalate with the overall rate of inflation assumed by Mr. Seiple.

Chapter Review

As with the previous chapters, I review some of the key points in the chapter:

Electricity Options Cannot be Structured in the Same Manner as Options on Other Storable Commodities: European options that can only be exercised at expiration do not make sense for a commodity that cannot be stored. In the case of electricity and other things that do not maintain value after expiration of the option, the most relevant option structure is one where exercise of the option can be repeated over and over again until the option expires. This Compound structure is unlike any of the option contracts used in financial securities, commodities or foreign exchange. Financial engineering must be used to build an option structure applicable to electricity from options on financial instruments.

Options Pricing Models Built on Constant Volatility Result in Biased Valuation if Applied to Electricity Prices: The volatility of electricity varies between on-peak and off-peak periods and is different for different seasons. Because of very high mean reversion in electricity, the standard deviation of electricity does not increase with longer and longer time periods. This means that volatility decreases for annual periods relative to monthly periods and for monthly periods relative to daily periods. The decline in volatility of electricity contrasts with the prices of many financial prices that follow a random walk and have constant volatility.

The Black Scholes equation and other option pricing models are built on the presumption that volatility does not change with different time periods. Because electricity prices are mean reverting and electricity prices do not follow a normal distribution, the necessary assumptions in applying the Black Scholes model do not exist for electricity. Even when the Black Scholes model is used to compute option prices on a short-term basis, the model does not approximate the true value of an option. However, the model can be useful in forward price models to compute the implied volatility of oil and natural gas from financial data.

Application of Monte Carlo Simulation to the Alternative Mathematical Formulations Can Be Used to Value Real Options: Monte Carlo simulation can be extended to value real options inherent in long-term assets. Through estimating volatility of annual prices, forward price levels, mean reversion and other factors, various types of real options can be valued. To value these options it is necessary to use judgement in establishing time series equations and analytical models in estimating time series parameters.

Complex Option Pricing Models are Unnecessary in Valuation of Peaking Plants: The value of options

on price series such as electricity with mean reversion and non-constant volatility can be estimated using Monte Carlo simulation. When formulas that are the basis for the Black Scholes model – i.e. random walk and normal distribution – are simulated using Monte Carlo simulation, problems with applying the Black Scholes model become obvious. An alternative approach to valuing peaking capacity is defining a jump process using statistical concepts. This allows one to evaluate the sensitivity of peaking capacity value to parameters such as the probability that price jumps occur, the standard deviation of prices and the level of price jumps.

Appendix 5-1
Applicability of Black-Scholes to Electricity

Because of the assumptions inherent in the Black-Scholes formula, direct application of the formula with no adjustments can lead to distorted results. For example, according to the publication "Institutional Investor", traders using Black-Scholes models mis-price over-the-counter electricity options. In an article on Black-Scholes, the author asserts, "some arbitragers using closed form models are reportedly picking off known Black-Scholers in the OTC electricity options market". The publication quoted the following analysts:

Julian Barrowcliffe, managing director at Cinergy Capital and Trading in Greenwich, Conn.

> "The Black-Scholes model was created with far less volatile instruments in mind. Black-Scholes is built on the assumption of lognormal distribution, constant volatility and the ability to gamma hedge continuously as delta moves. For example, the market is not supposed to gap. None of these assumptions holds true for the OTC electricity options market."

John Wengler, v.p. at SAVA Risk Management:

> "Prices in the electricity market tend to show a stronger tendency to revert to their mean than most assets. For this reason, a trader using a model that does not assume constant volatility to price a one-month electricity option when implied volatility is at 300%, would be able to sell the instrument to a Black-Scholer at this level, knowing that the implied volatility would likely mean revert to a lower level."

Carolyn Pangidore, director-energy risk management at Arthur Andersen:

> "Traders need to continuously test and recalibrate models as markets change. Model arbitrage is not a panacea. It's pretty dangerous. I'd call it a high-risk strategy."

Appendix 5-1
Applicability of Black-Scholes to Electricity

Use of a Risk-free Rate in the Black-Scholes Formula

The method for valuing companies using a "risk-neutral" method that is described in the next chapter is applicable to the Black-Scholes equation because none of the variables in the formula are affected by the risk adjusted cost of capital (i.e. traditional risk versus return tradeoffs). The risk neutral approach has become popular to consider many valuation problems, where construction of two portfolios with the same cash flows can be used to create risk free cash flows. If the two portfolios have the same cash flows and one of the cash flows is shorted, then the two portfolios must have the same value. If we know the value of one portfolio, then we can compute the value of the other value using the risk free rate as the discount rate.

Option values are not computed using a discount rate that includes risk, but rather contractual cash flows discounted at a risk free rate. The variables that do appear in the equation are the current stock price, time, stock price volatility, and the risk-free rate of interest. All of these variables are independent of the risk adjusted cost of capital that drives the expected growth in stock prices. The Black-Scholes equation would not be independent of risk preferences if it involved the expected return on the stock because the value of expected rate of return does depend on risk preferences.

In financial economics, there is a positive relationship between expected return and risk preference of investors. The higher the level of risk, the higher will be the expected rate of return. In the Black-Scholes model, variables for the expected return drops out of the formula in derivation of the equation. This derivation uses an arbitrage approach where two portfolios are created, one with a derivative security and one with the security itself and risk free bonds. With the correct mixture of securities and bonds, the two portfolios have the same cash flows. Option pricing is derived by assuming one of the portfolios is held long and the other is shorted. When the long and short portfolios are combined, the net cash flow is risk free. Whenever a portfolio can be replicated from other securities risk free cash flows are created, the risk free rate is appropriate. The growth in stock price from the expected return necessary to compensate for risk is off-set by the increased risk associated with future cash flows. As John Hull points out, one can make the very simple assumption that all investors are risk neutral.

Properties of the Black Scholes Formula in Extreme Situations

I find it instructive to demonstrate how the Black-Scholes model works in practice by considering what happens when some of the variables become very large or very small. In these extreme situations, option prices are straight forward on a conceptual basis and one can see that the Black-Scholes model in fact yields logical results. I first address high or low prices and then low volatility. These extreme examples can be verified by working with the spreadsheets described in section five of the workbook.

High or Low Price

When the price (the stock price, bond price, commodity price, or in our case the electricity price) reaches very high levels – far above the exercise price -- a call option (allowing purchase at a given exercise price) is virtually certain to be exercised. In different parlance, the option is "in the money" by a wide margin. The option then becomes very similar to a forward contract where a the financial security or a commodity must be purchased at a set price at a future date. In this case, the call price of the option should be approximately the same as the result of the formula below, which corresponds to the price of a forward contract:

Call Price of Option = Stock Price - PV of Exercise Price

Intuitively this formula means the current price discounted for carrying cost is the correct reflection of today's value of a forward contract. Assuming a zero interest rate, the option price is simply the stock price minus the exercise price. If the forward contract has any other price, arbitrage can be used by borrowing money and holding the stock.[14] Since the call option is a European option, one does not exercise it until the expiration date – analogous to time of expiration in settling a forward contract. The resulting call price (the stock price less the present value of the exercise price) in the above formula is given by Black-Scholes equation because the formulas for both d_1 and d_2 include the factor current price/exercise price. If d_1 and d_2 are very large, then the normal distribution for d_1 and d_2 are both close to 1.0 and the formula reduces to the specification shown above.

Low Volatility

When the volatility approaches zero, an investment is virtually riskless and the price of a security should grow at the risk free rate. In this case, the payoff from a call option is given by the formula:

Payoff = Greater of [Current Price - PV of Exercise Price, or Zero]

In the Black-Scholes formula, as volatility reaches zero, d_1 and d_2 approach □ because volatility is in the denominator of the equations for d_1 and d_2. The large values of d_1 and d_2 suggest that the normal distribution of d_1 and d_2 is about 1.0. Substituting large values for d_1 and d_2 into the Black-Scholes formula results in different outcomes depending on whether the price is above or below the exercise price. If the current price is more than the present value of the exercise price, the Black-Scholes equation becomes:

Call Price = Current Stock Price - PV of Exercise Price

The above equation is intuitive because if there is there were no volatility, the current price grows by the risk free rate until expiration of the option. If the risk free rate is zero, the call price simply measure how much the option is in the money. If the risk free rate is positive, the option is worth more because the stock price is expected to grow at the risk free rate while the exercise price is constant. In other words, through purchasing a call option, you receive the future price and pay the exercise price. The option will be always be exercised and the price grows at the risk free rate.

In the case where the stock price is less than the present value of the exercise price, the call option has no value in the zero volatility case. As volatility approaches zero, d_1 and d_2 tend to -□ so that $N(d_1)$ and $N(d_2)$ move close to zero. In this case, the Black-Scholes equation gives a call price of zero. Again, this is logical, since if the exercise price is above the current price and there is no chance for the price to move above the exercise price, the call option is worthless.

Twenty years ago, prices on the vast majority of electricity transactions between utility companies in the United States were regulated and relatively few trades occurred. Today, competitive wholesale markets in electricity generation have blossomed all over the world. Daily (and in some cases hourly) market prices are regularly published and a number of trades are made in forward markets that lock-in future prices. A wide range of complex transaction structures have emerged for trades of electricity including forward price contracts and

14 If the borrowed money is used to buy the stock, and a forward contract is sold, the holder can sell the stock, repay the loan and receive payment for the forward contract.

options. Arguably, electricity has become a commodity analogous to gold, grains, oil and natural gas. Before it went bankrupt, Enron grew from a traditional pipeline company to the seventh largest corporation in the United States largely through trading electricity and natural gas. Behind the "house of cards" that caused Enron's demise was un-hedged exposure to the high volatility of electricity and natural gas prices.

The Black Scholes Model and Compound Option Valuation

In this section, the Black-Scholes model is used to measure an option that can be exercised repeatedly that mimics the dispatch of a generating plant. Finally, Monte Carlo simulation is used to value the same option contract that replicates the dispatch option of a plant discussed above. Once completing the analysis of option valuation for electricity contracts, the efficacy of the option models in pricing electric capacity is considered.

The option structure is a contract with a Compound exercise rather than the single exercise at expiration in the European option structure. Recall that the Black-Scholes model derives the value of a European option that is exercised at one time at the expiration of the option. A peaking plant that is representative of the pricing of capacity does not have this structure. Instead, peaking capacity is analogous to a set of multiple options as discussed above. The structure of this option is one where the buyer can exercise the option during any **hour** prior to expiration; and once the option is exercised, it can be re-exercised at any time when the electricity price exceeds the strike price.

To assure that capacity will be available for the entire year, 252 options must be purchased.

To evaluate different models using parameters from electricity price series and an option structure that represents peaking capacity, consider a one-year option using three different models. These models include:

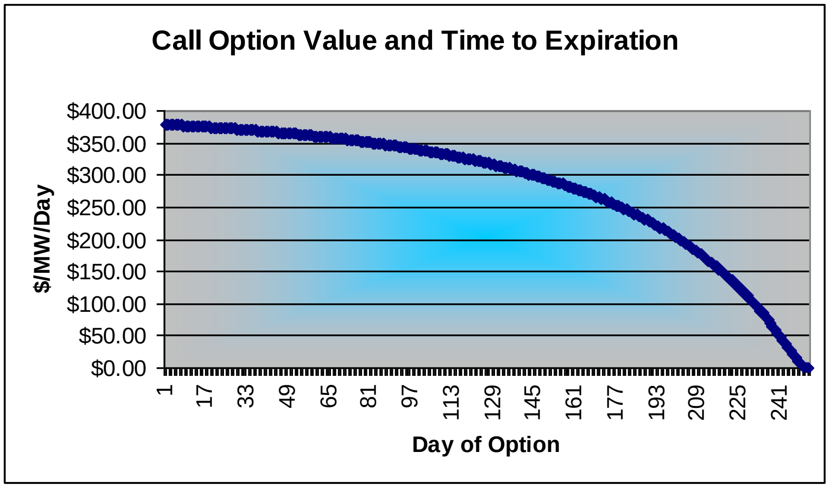
-The Black-Scholes model with electricity price volatility.

-Monte Carlo simulation without mean reversion.

-Monte Carlo simulation with mean reversion.

Using the Black-Scholes model to electricity prices involves inputting a high level of volatility into the equation and adjusting the model to account for the Compound structure. The major difference between applying the model to electricity and employing it to value a financial security involves adjusting the model for the Compound structure and converting the option prices expressed in terms of $/MWH to capacity values expressed in $/kW/year. Even though the Black-Scholes has significant problems, working through the Compound option and the unit conversion is instructive for many applications.

To demonstrate application of the Black-Scholes model to electricity assume an option that can be exercised on a daily basis every day for a year and that it can be re-exercised each day. Conversion of the option using daily prices expressed on an average $/MWH basis to an annual capacity charge on a $/kW/year basis using the Black-Scholes model involves the following four steps:



Step 1: Convert options stated in hourly units to daily options measured on a $/kW/day basis.

Step 2: Re-compute 252 daily options with gradually declining expiration dates.

Step 3: Sum the value (on a present value basis) of the daily options to establish the value of the annual option.

Step 4: Convert options stated in daily terms to an annual capacity charge.

The option contract gives the holder a right to use power for a specified period of time longer – a day -- than an hour which is the basis for pricing. The $/MWH prices must therefore be converted to a daily period through multiplying the $/Megawatt-hour number by the number of on-peak hours in a day -- 16 hours. Once this conversion is made, all prices -- the current price, the exercise price and the prices used for computing volatility should be measured in terms of $/MW/day. Assuming current prices are $35/MWH and the exercise price is $40/MWH implies a current daily price of $560/MW/day and an exercise price of $640/MW/day.

Since the option is repeated, the value of 252 options must be computed and summed. The sum of the option values results in an amount expressed in $/MW/day x days/year or $/MW/year. Dividing by 1000 yields units of $/kW/year. Each daily option has a different value because there is less and less time until the option expires. The daily value of the options are shown on the accompanying graph. In our example, the annual cost of the option at a strike price of $60/MWH is $70,560/MW/Year (the area underneath the graph). This is the sum of daily options on the accompanying graph. If it is assumed that purchasing the option gives the holder the right to use energy for the full day, the capacity price is $70.56/KW/Year.

To evaluate the option pricing, one can compute how many hours would be exercised on a daily basis using actual market price data. The value of electricity above $60/MWH on a daily basis was $11,733 summed over all the days for five years. Converting the value to an annual basis yields an annual value of only $37.54/KW/Year ($11,733 / 5 x 16/1000). The significant difference between option values using the two approaches -- $70.56 versus $37.54 demonstrates problems with use of the Black-Scholes model in situations with very high volatility, a non-normal distribution and mean reversion.

Monte Carlo Simulation Applied to a Valuation of a Compound Option

The value of the Compound option in Monte Carlo simulation is computed differently from the European option described above in the calibration process. Instead of sorting the prices in the last period and comparing prices for that day, an “if test” is performed each day to derive the option payoff.

For each day, if the price for that day is above the exercise price, the payoff value is aggregated. Since the test is made on a daily basis and prices are in hourly units, the daily payoff must be multiplied by the number of hours in the daily period – 16 hours.

The daily payoffs are then aggregated for the year to establish capacity value in $/kW/Year. Using this process with the time series where prices cannot be negative results in an option value of $80/kW/Year.

The Monte Carlo simulation demonstrates that the Black-Scholes equation is very flawed in estimating option values for electricity.

> PARKED
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> For most of us, the discussion of volatility, option value and risk adjustments that once occurred in meetings with Enron executives may have been somewhat daunting. Unfortunately, the trading concepts and terminology – swaps, puts, calls, implied volatility, strips and so forth -- are here to stay despite the fact that Enron executives are no longer selling products.

Analysis of option pricing in the context of generating plant analysis adds a risk dimension to the forward pricing models of the last two chapters because it focuses on the dispersion in future prices and the time series process of electricity. The discussion of option modeling in this chapter is focused on the variation around expected prices.

Alternative Terms with Respect to Exercising an Option over the Duration of a Contract

The statement that operation of a power plant is analogous to an option has become a common axiom in the electric generating industry. However, the details of option pricing contracts can significantly affect their value and if power plants can be represented as option contracts, the terms of that option must be defined. Contractual terms of an option include: (1) the strike price, (2) the length of time over which the option can be exercised, (3) the manner in which the instrument will be delivered if the holder decides to exercise the option and (4) definition of how the option can be exercised during the duration of the option. Some of the implications of the options analysis include:

(1) Options contracts in electricity cannot have exercise provisions structured as a European option or other common structures used in contracts for options on financial securities;

(2) Since electricity prices do not follow a random walk or have constant volatility, analytical option pricing models such as the Black-Scholes formula do not appropriately value option contracts on electricity price;

(3) Because of very high mean reversion in price spikes for electricity and because peaking plants derive most of their value during price spikes, option valuation models that are derived from time series analysis of prices are not of much use in valuation of electricity plants; and,

(4) Monte Carlo simulation can be used with time series equations to value real options inherent in long-term assets such as the obligation to be a provider of last resort through estimating time series parameters described in Chapter 2 such as long-term volatility and mean reversion.

The strike price of the dispatch option is the variable operating cost of the plant. The length of time until expiration can vary from an hour to the life of the plant, but can be objectively defined depending on the valuation analysis. Delivery provisions are a function of the location of the plant and transmission constraints. While these terms can be clearly defined for the option to dispatch, the definition of how the option can be exercised does not correspond to classic option contracts. The remainder of this section deals with how exercise provisions affect value in the context of electricity options. Three issues related to structuring the exercise provisions of an option include:

- How do differences in economic characteristics between electric energy and financial securities – the lack of storage, mean reversion and price jumps -- affect the plausible terms of an option contract?

- How can realistic exercise provisions of electricity options contracts be developed to account for price movements that arise from the lack of storage of electricity?

- How do the "mean reverting" provisions of electricity as compared to the "random walk" characteristics of stock prices affect analytical option pricing models?

Option contracts on financial securities have three general structures -- (1) a **European Option** which can only be exercised at expiration; (2) an **American Option** that can be exercised at any time prior to the expiration date; and. (3) an **Asian Option** that is based on average prices rather than instantaneous prices. Because the price characteristics of electricity differ from financial securities and other commodities, the specific terms of electricity options contracts can have a very significant impact on value. On the other hand, in the case of options on financial securities, the type of contract has less impact on the price of the option.

The Effect of Option Contract Structures on Financial Securities

To see that the option structure may not have a large impact on financial securities, consider the case of a stock that does not pay dividends and where prices are distributed in a lognormal manner without mean reversion. In this situation, it can be demonstrated that the value of an American option is the same as the value of a European option.[15] To demonstrate this, consider two call options on a stock that are both "in the money" (i.e. where the price is below the strike price). The two options have the same contract terms except that one is an American option and the other is a European option. Since the option is in the money, the holder of the American call option could exercise the option and receive an immediate profit. Alternatively he can hold on to the option and try to receive even more profit in case the stock price increases further. The option has more value than if it were sold because the upside from prices potentially moving even higher is less than the downside that is limited by the ability not to exercise the option. If the holder of the option does not exercise the option until its expiration, the European option and the American option have the same value.

15 See Chapter 10 of "Options, Futures and Other Derivatives" by John C. Hull. Prentice Hall, N.J., 1997.Hull and White.

This is demonstrated by considering three alternatives for the holder of the American option:

1) Exercise the option and receive profit from buying the stock at the strike price. In this case, the holder gives up potential future profits if the stock price moves up further.

Profit = Current Price – Exercise Price

2) Sell the option. Since the option price reflects the difference between stock price and exercise price as well as potential for increased value, the value of this alternative exceeds the value of selling the option in alternative 1.

Profit = Valuation of Option to a Potential Buyer, where

Buyer Value = Current Price – Exercise Price + Value of Option with Exercise Price Equal to Current Price

3) Hold on to the option and exercise it at maturity. The value of this option is the same as 2) above.

Since the profit from holding the American option until expiration exceeds profit from exercising earlier at the strike price, there is no economic benefit to early exercise and the provision in the American option for early exercise has no value. The price and value of an American option should be the same as the price and value of a European option. Demonstration that an American option and a European option have the same value if the option is not in the money is easier. If an option is "out-of-the money," exercising the option gives you nothing, so you might as well wait until the last moment that you can exercise it whether the contract specifies that it can be exercised early or not.

If a stock pays dividends, the value of the stock declines when the dividends are paid. This means the price pattern of the stock price does not follow a pure random walk process and the prices do not simply wander up. Valuation of the option must reflect the expected stock price movement adjusted for the decline cause by the dividend. While dividends can significantly affect valuation of an option contract, since dividends are known, reasonable valuation adjustments can be developed and the analytical process remains similar.

Possible Option Contract Structures Applicable to Electricity Prices

While exercise provisions of an option on a financial security do not have a major impact on option value, the same is not true for an option on electricity. Because electricity cannot be stored and because electricity prices revert to average levels, the contract provisions of electricity affect option value in a different manner than the terms affect other commodities. To demonstrate how electricity options are affected by exercise provisions, consider the following five different option structures in the context of electricity.

A European Option than can only be exercised at the expiration date. For example if the strike price is $35/MWH and the option contract gives the holder a right to strike for one hour, the actual price must be above $35/MWH in that hour of expiration for the option payoff to be positive.

An ***American*** Option that can be exercised at any time prior to maturity; but once exercised the option holder must take power. For example, if the exercise price is $35/MWH and the option is exercised, the holder must take power for the duration of the option even if the price subsequently falls.

An ***Asian*** Option that is exercised according to an average price level rather than a single hourly price level. For example, an option holder could buy the right to receive energy at an average level of $45/MWH in August.

A ***Daily or Weekly*** Option that can exercised on a weekly on-peak basis where the holder can strike at the beginning of a week by accepting five days of power for 16 hours a day (a 5 x 16 matrix). The holder of the option can re-exercise on a weekly basis for each week of the remaining term until expiration.

A ***"Compound"*** Option that can be exercised on an hourly basis at any time prior to expiration. Once the option is exercised, it can be exercised again, but the holder is not required to take electricity for the duration of the option. This structure is analogous to the operation of a fast start peaking plant. The structure is also analogous to a series of European options, each with a different expiration date. For a year, the Compound option is equivalent to 8760 European options with 8,760 different expiration dates.

European Options Applied to Electricity

The fact that options on electricity have values that depend on the contract provisions is demonstrated by the difference in value between the structures defined above. Consider the difference between the European option exercisable at expiration and option contract where the holder has an ability to exercise at any hour. For a given strike price, the European option probably has very little value compared to structure where the holder can strike in any hour. In the case of the European option, the potential payoff depends on weather conditions, plant outages, hydro conditions and many other factors at a single point in time when the option expires.[16] On the other hand, in the option structure where the option can be exercised at any hour, the option may be exercised at expiration along with any other time during the term of the option.

The ability to exercise in any hour implies that the Compound option structure has many multiples of value as compared to the basic European option. Indeed, because the value of electricity goes away as time passes -- a kilowatt hour has no value once the hour has passed -- the pure European option makes little sense in the context of electricity. In the case of a European option, the holder receives value for one hour (or some other period a day or a week around expiration) but has no rights to anything of value after the term of the option. Contrast this to natural gas, where an MMBTU of gas can be purchased through exercising an option can be stored, used, or sold at some time in the future.

American Options Applied to Electricity

The structure of an American option seems to have more relevance to electricity than the European option structure, but also may not be realistic in practice. In the case of an American option, as soon as the option is exercised, the holder of the option must take electricity at the strike price. Once the option has been exercised the electricity is generally held for the remaining duration of the option. If prices are high in an hour when the option is exercised and then prices decline while the option is being held, exercising the American option may produce a negative payoff. For example, if the option can be exercised at $40/MWH and the price increases to $100/MWH for five hours, the holder profits from exercising his right to buy power for those hours. However, if the price subsequently declines to $20/MWH, the option holder takes a loss through having

[16] Later in the chapter we describe how the European option can be adjusted to reflect average prices rather than hourly prices.

to buy at \$40/MWH when the price is \$20/MWH. This does not occur in the case of a financial security where once the option has been exercised, the holder can re-sell the stock and have no financial exposure to potential price declines.[17]

Spark Spreads and Compound Options

Given the problems with traditional option structures to electricity, I focus much of the analysis in the remainder of this chapter is on the "Compound" option discussed above. From the perspective of capacity pricing, the Compound option structure comes closest to mimicking capacity where the hourly strike where the strike price is the marginal energy cost. This contract structure is analogous to owning and dispatching a power plant. When prices exceed the marginal energy cost, the option is exercised and the call price represents the amount that should be paid for capacity. While this Compound option with an hourly strike price structure is most relevant to electricity pricing and valuation analysis, it is not consistent with option pricing models such as the Black Scholes model. These models have been developed for European (and to some extent American) options and must be adjusted.

Problems with a changing strike price arising because of varying marginal costs (driven by changes in the natural gas price) can be dealt with by applying the option to a spark spread rather than an absolute price. In this case, the price in the option is the difference between the gas price and the electricity price rather than the absolute gas price. Volatility and all of the option terms can be defined for the spark spread instead of the actual electricity price.

Spark Spread/MWH = Electricity Price/MWH – Natural Gas Price x HR,

Where HR is generally between 7 and 10 depending on the type of plant or contract being analyzed and the Natural Gas Price is expressed in MMBTU

Without storage costs, the value of a forward contract or future is less than the current spot price. This is because one can borrow money and buy the item (say the oil price) and this strategy has identical payoffs to the value of the future. For example, if the current oil price is 100, the future contract is for a year and the storage cost is zero, then you can buy oil, put it in a barrel and at the end of the year sell the oil. Alternatively, you could sell a futures contract for the oil and when the contract reaches its maturity, the value of the future should be the same as the value of buying the oil. In this case, you would have to borrow money to buy the oil, meaning that if the two strategies produce the same value, then the futures contract is simply expressed as:

Value of Future = Current Spot Price x e^{-rt}

In this equation r is the risk free rate and t is the term to expiration of the forward contract. If you hold an option on a futures contract, then you would not have to buy the present value of the spot price rather than present value of the cash flow.

[17] It could be possible to sell contracts for electric power for the duration of the option, however, it is questionable whether this could be accomplished without very high transaction costs.